Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351201_OC**
Name of Paper	: **C 3-Real Analysis**
Semester	: **II**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

Q1. Find the infimum and supremum, if they exist, of the following subsets S_i $(i = 1,2,3)$. Justify your answer in each case:

$$S_1 = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$$
$$S_2 = \left\{\frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$$
$$S_3 = \left\{\frac{1}{n} + 1 : n \in \mathbb{N}\right\}$$

Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a \in \mathbb{R}$ and define a set $a - S = \{a - s : s \in S\}$. Prove that $a - S$ is a bounded set and $\sup(a - S) = a - \inf S$.

Q2. Let $S = \{s \in \mathbb{R} : 0 \leq s \ \ and \ s^2 < 3\}$. Show that the set S has a supremum in $\mathbb{R}$. If $x = \sup S$, prove that $x > 0$ and $x^2 = 3$. What is $\inf S$?

Let u and v be real numbers with $u < v$. Show that there exists a rational number r such that
$$u < \sqrt{3}\, r < v.$$

Q3. Discuss convergence or divergence of the following sequences. If convergent, find the limit of the sequence (x_n) using ϵ- definition and if divergent, give reason for the same:

(i) $x_n = \frac{2n+3}{3n+7}$ (ii) $x_n = \frac{n}{(1-n)(1+n)}$ (iii) $x_n = \frac{2^n+4^n}{3^n}$

Are these sequences bounded? Justify your answer in each case.

If (x_n) is a convergent sequence with $x_n \geq 2$ for all $n \in \mathbb{N}$, prove that $\lim(x_n) \geq 2$.

Q4. Using the definition of Cauchy sequence, establish the convergence or divergence of following sequences

(i) $\left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots - \cdots + \frac{1}{n!}\right)$ (ii) $(\ln n^2)$ (iii) $((-2)^n)$

Show that a sequence (x_n) defined as
$$x_1 = 1$$
$$x_{n+1} = \frac{x_n + 3}{5}, \qquad n \geq 1$$
is convergent and find its limit.

Q5. Check the convergence or divergence of the following series. Clearly specify the result being used:

(i) $\sum_{n=1}^{\infty} e^{-n^2}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{\log n}$

(iii) $\sum_{n=1}^{\infty} \frac{n+1}{2^n}$

(iv) $\sum_{n=1}^{\infty} \left(\frac{n}{n-1}\right)^{n^2}$

Q6. For each of the following, determine whether the series converges absolutely, converges conditionally, or diverges.

(i) $\sum_{n=1}^{\infty} \frac{2^n + n}{2^n - n}$

(ii) $\sum_{n=1}^{\infty} \frac{(\sin n\alpha + cos^2 n\alpha)}{n^2}$

(iii) $\sum_{n=1}^{\infty} \frac{(-1)^n \, 100^n}{n\,!}$

(iv) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \log n}$

[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : **Your Roll No………**

Unique Paper Code :32351202_OC

Name of the Course : **B.Sc. (Hons.) Maths-I**

Name of the Paper : C4-Differential Equations

Semester : II

Duration: 3 Hours Maximum Marks: 75

Instruction for Candidates

1) **All questions carry equal marks.**
2) **Attempt any four questions.**

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1. (a) Find a general solution of the following differential equations.

$$\frac{dy}{dx} = \sqrt{x + y + 1}$$

(b) The half -life of a radioactive cobalt is 6.82 years. Suppose that a nuclear accident has left the level of cobalt radiation in a certain region at 100 times the level acceptable for human habitation. How long will it be until the region is habitable?

2. (a) Use the method of variation of parameters to find a particular solution of the differential equation $y'' + 4y = x$.

(b) Find the general solution of $(4xy') + y^3 e^{-2x} = 4xy$

3. (a) A water tank has the shape obtained by revolving the curve $y = x^{8/3}$ around y-axis. A plug at the bottom is removed at 1:00 P.M., when the depth of the water in the tank is 18 ft. At 3:00 P.M. the depth of the water is 9ft. When will the tank be empty?

(b) Find the general solution of $xy'' = y'$.

4. (a) Let R(t) denote the number of red army and B(t) denote the number of blue army. Assuming both the armies use aimed fire, formulate the model (a pair of differential equations) and solve them to find the general solution. Also develop a model (a pair of differential equations) for a battle between two armies where both the groups use aimed fire. Assuming that the red army has significant loss due to disease, where the associated death rate (from disease) is proportional to the number of soldiers in that army.

(b) Find the solution of $x^2y'' - 6xy' + 3y = 0, y(2) = 3, y'(2) = 1$.

5. (a) A mass of 5 kg is attached to the end of a spring that is stretched 25 cm by a force of 18 N. It is set in motion with initial position $x_0 = 0$ and initial velocity $v_0 = -15\ m/s$. Find the amplitude, period and frequency of the resulting motion.

(b) Use the method of undetermined coefficients to find the general solution of

$y'' - 4y' + 4y = e^{2x}$

6. (a) In a poultry farm, hens are harvested at a constant rate of 700 hens per day. The per-capita birth rate for the hen is 1.4 hens per week per hen, and the per-capita death rate is 4.9 hen per week per hen. Defining each symbol you introduced, write the word equation to describe the rate of change of hen population. Using the above written word equation, obtain the differential equation describing the rate of change of hen population. If the hen population at a given time is 280000. Estimate the number of hen died in one week. Determine if there are any values for which the hen population is in equilibrium.

(b) Show that the differential equation $(4x + 3y^2)dx + 2xy\ dy = 0$ is not exact. Find an integrating factor & solve.

Name of the course : **CBCS B.Sc(H)Mathematics**

Unique Paper Code : **32357609**

Name of Paper : **DSE-3 : Bio-Mathematics**

Semester : **VI**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Describe the mathematical model of population growth for any species changing by birth only. Draw and describe the graph of the behavior of the population with an increase in time. What would be the change in your model if you consider deaths of individuals also along with births. In a research on population dynamics of mosquitoes, it was estimated that the initial population is 2000. Over the time period of one month, 300 births and 100 deaths were recorded in the population. Predict the population size at the end of 10 months.

2. Find and draw the trajectories in the phase plane of

a) $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0.$

b) $\frac{dx}{dt} = 5x + 2y$; $\frac{dy}{dt} = -2x + 5y$.

3. What is the threshold effect in a heartbeat model? Why is it an important feature to be included in the model? Modify the following model for heartbeat to reflect the threshold effect.

$$\epsilon\frac{dx}{dt} = -a(x - x_0) - (b - b_0)$$

$$\frac{db}{dt} = x - x_0$$

where (b_0, x_0) is the unique rest state. Present a phase plane analysis of the model obtained above and explain how it includes the physiological considerations of the heartbeat cycle?

4. Define Hopf bifurcation. Show that the system

$$x' = -y + x(\beta - x^2 - y^2)$$

$$y' = x + y(\beta - x^2 - y^2)$$

has Hopf bifurcation. Consider the iteration scheme for points on a Poincare Plane:

$$x_{n+1} = \frac{1}{2}y_n$$

$$y_{n+1} = -x_n + \frac{1}{2}\alpha y_n - y_n^3$$

Show that there is a bifurcation at $\alpha = 3$. Show that the limit cycle in $0 < \alpha < 3$is stable and becomes of saddle type when αexceeds 3.

5. Consider ancestral and descendent sequences of 400 bases which were simulated according to Kimura 2-parameter model with $\gamma = \frac{\beta}{5}$.A comparison of aligned sites gave the frequency data in Table-1 below:

Table -1

S_1/S_0	A	G	C	T
A	92	15	2	2
G	13	84	4	4
C	0	1	77	16
T	4	2	14	70

Compute the Jukes-Cantor distance and Kimura 2-parameter distance to 10 decimal digits.

In what cases Jukes-Cantor distance is zero? Derive the formula $d_{JC} = -\frac{3}{4} ln(\frac{4q-1}{3})$ where q is the proportion of bases that are the same in the before and after sequences.

6. Draw the single topologically distinct unrooted bifurcating tree that could describe the relationship between 3 taxa. Draw Punnett square for $Dd\ X\ Dd$ and $DdWw\ X\ ddWw$, where D denotes allele for dominant plant, d allele for dwarf plant, W dominant allele for round seeds and w recessive allele for wrinkled seeds. What is the genotypic ratio and phenotypic ratio in $Dd\ X\ Dd$? What percentage is the progeny of $DdWw\ X\ ddWw$ dwarf with round seeds? Now consider the distance data in the following Table-2, which is exactly fit by the following tree, use UPGMA to construct a tree from this data. Also use Neighbor-Joining method to compute R_1 , R_2 , R_3 and R_4 , and then a table of values for M for the taxa $S1$, $S2$, $S3$ and $S4$.

Table-2

	$S1$	$S2$	$S3$	$S4$
$S1$		0.3	0.4	0.5
$S2$			0.5	0.4
$S3$				0.7

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351601**
Name of Paper	: **C 13- Complex Analysis**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine whether $S = \{z \in \mathbb{C} : |z|^2 > z + \overline{z}\}$ is a domain or not? Justify your answer.

Find the image of line segment joining $z_1 = -i$ to $z_2 = -1$ under the map $f(z) = \overline{iz}$.

Check whether Cauchy-Riemann equations for $f(z) = \sqrt{|z^2 - \bar{z}^2|}$ are satisfied at the origin? Is f analytic at the origin? Justify your answer.

Suppose $f(z) = \cosh(2x)\cos(2y) + i\,v(x,y)$ is analytic everywhere such that $v(0,0) = 0$. Find $f(z)$. Hence find zeros of f.

Solve the equation $e^{z-1} + ie^3 = 0$.

2. Let $S = \{z \in \mathbb{C} : \text{Im } z = 1 \text{ and Re } z \neq 4\}$. Is S open? Is S closed? Justify your answer.

Assume that g is analytic in a region and that at every point either $g = 0$ or $g' = 0$. Show that g is constant.

Suppose $f(z) = \begin{cases} \bar{z}^3/z^2 & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$. Show that f is continuous everywhere on $\mathbb{C}$. Is f analytic at $z = 0$? Justify your answer.

Does there exists an analytic function $f(z) = u(x,y) + i\,v(x,y)$ for which $u(x,y) = y^3 + 5x$? Solve the equation $Log(z) + Log(2z) = 3\pi/2$.

3. Determine whether the following curves are simple, closed, smooth or contour

C_1: $z(t) = |t| + it$, $t \in [-1,1]$

C_2: $z(t) = e^{2it}$, $t \in [0, 2\pi]$,

C_3: $z(t)$ is the positively oriented boundary of the rectangle whose sides lie along $x = \pm 1$, $y = 0$, $y = 1$.

Evaluate $\int_{C_3} |z|dz$. Explain why Cauchy Goursat theorem is not applicable in this case?

Use ML-Inequality to show that

$$\left|\int_C \frac{e^z}{(z+1)}dz\right| \leq 4\pi e^2$$

where C : $z(t) = e^{2it}$, $t \in [-\pi, \pi]$.

4. Evaluate $\int_C ze^{3z}dz$ where C is the parabola $x^2 = y$ from $(0,0)$ *to* $(1,1)$.

Using Cauchy Integral formula, determine the integral $\int_C \frac{e^z}{z^2(z^2-9)}dz$ where C is positively oriented circle (i) C: $|z| = 1$. (ii) C: $|z - 3| = 1$.

Use Liouville's theorem to establish that $\cos z$ is not bounded in the complex plane.

Let g be an entire function and suppose that $|g(z)| < 10$ for all values of z on the circle $|z-2| = 3$. Find a bound for $|g'''(2)|$.

5. Determine the radius of convergence of the series $\sum_{k=0}^{\infty} \frac{z^k}{k!}$ and $\sum_{k=0}^{\infty} k^k z^k$. Also discuss the convergence of the series.

Obtain the Maclaurin series of the function (z) $= \frac{1}{z^2} sinh\left(\frac{1}{z}\right)$. Specify the region in which the series is valid.

Find the Laurent series of the function $f(\text{z}) = \frac{1}{(z+1)(z+3)}$ valid for $0 < |z+1| < 2$.

6. Determine the residue and singularities of the function $g(\text{z}) = \frac{z+1}{z^2+4}$. Also evaluate $\int_C g(z)dz$ where C is the positively oriented circle $|z-i| = 2$.

Using a single residue, evaluate the integral $\int_{C'} \frac{3z-1}{z(z+1)} dz$ where C' is the positively oriented circle $|z-1| = 4$.

Use residue to evaluate the integral $\int_0^{2\pi} \frac{dt}{3+cost}$

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357611**
Name of Paper	: **DSE-4 Linear Programming and Theory of Games**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

Q.1 Solve the following LPP by Big-M method and verify your answer by finding all the existing basic feasible solutions:

Maximize $Z = x_1 - x_2 - x_3$

Subject to $x_1 + x_2 + x_3 \geq 2$

$2x_1 - x_2 + x_3 = 3$

$x_1, x_2, x_3 \geq 0$

Q.2 Obtain the inverse of the following matrix by using simplex method

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

Verify your answer by matrix multiplication.

Q.3 Verify for the following Linear Programming Problem that dual of dual is primal. Also using complementary slackness theorem solve both primal and dual problems.

Maximize $Z = x_1 + x_2$

Subject to

$x_1 + 2x_2 \leq 5$

$2x_1 + x_2 \geq 0$

$x_2 \leq 3$

$x_1, x_2 \geq 0.$

Q.4 For the following cost minimization transportation problem find initial basic feasible solutions by using North West Corner rule, Least cost method and Vogel's approximation method. Compare the three solutions:

Source \ Destination	A	B	C	D	E	Supply
I	16	16	13	22	17	50
II	14	14	13	19	15	60
III	19	19	20	23	15	50
IV	12	10	15	8	12	50
Demand	30	20	70	30	60	

Also find the optimal basic feasible solution of above problem using UV- method.

Q.5 Solve the cost minimization assignment problem:

Job \ Man	I	II	III	IV	V
A	2	3	5	5	6
B	4	5	7	7	8
C	7	8	8	10	9
D	3	5	3	6	5
E	4	3	5	2	1

Does this problem has more than one solution? If yes, then find any FOUR possible solutions.

Q. 6 Show that the following rectangular game does not have any saddle point.

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 12 & -6 & 3 & 0 \\ 4 & 0 & 2 & 1 \\ 0 & 4 & 3 & 4 \\ 6 & -1 & 3 & -2 \end{bmatrix}$$

Solve it by graphical method by reducing its size using dominance principle.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357610**
Name of Paper	: **DSE4: Number Theory**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Using the theory of linear Diophantine Equation, divide 299 into two summands such that one is divisible by 12 and the other by 17.

If p is an odd prime divisor of (n^2+1), then prove that $p \equiv 1 \pmod 4$.

Find all primitive Pythagorean triples x, y, z in which x = 20.

2. Solve the following set of simultaneous congruences:

$x \equiv -1 \pmod{27}$

$x \equiv -2 \pmod{16}$

$x \equiv 0 \pmod{25}$

Using the theory of congruences show that the sum $1^5 + 2^5 + 3^5 + 4^5 + \ldots\ldots\ldots\ldots + 100^5$ is divisible by 4.

3. Mobius pair is a pair of functions {f(n), g(n)} such that $f(n) = \sum_{d|n} g(d)$ where the sum runs over all positive divisors of the positive integer n. Prove that if one of the functions of the Mobius pair is multiplicative, then so is the other.

Using the Mobius Inversion formula, deduce that for all n>=1,
$\sum_{d|n} \mu\left(\frac{n}{d}\right) \tau(d) = 1$ and $\sum_{d|n} \mu\left(\frac{n}{d}\right) \sigma(d) = n$

4. Find the last two digits in decimal representation of 13^{1010} .

Find the sum of positive integers less than 1001 and relatively prime to 1001.

Also show that $\frac{(a+b)!}{a!\,b!}$ is an integer for any positive integers a and b.

5. Find all positive integers less than 37 having order 6 (mod 37).

Determine whether the quadratic congruence $x^2 \equiv -72 \pmod{131}$ is solvable.

Find all odd primes $p \neq 3$ having 3 as quadratic residue.

6. The ciphertext VKYAQ VAKEC has been enciphered with the Linear Cipher

$$C \equiv 17P + 10 \pmod{26}$$

Derive the plaintext. When the RSA algorithm is based on the key (n, k) = (2419, 11), what is the recovery exponent for the cryptosystem?