Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351101**
Name of Paper	: **BMATH101-Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Sketch the graph of the function

$$f(x) = 3x^4 - 4x^3$$

by finding the intercepts, critical numbers, intervals of increase/decrease, relative extrema, second-order critical numbers, concavity and inflection points.

It is projected that t years from now, the population of a certain country will be

$$P(t) = 50\, e^{0.02t} \text{ million.}$$

(i) At what rate the population is changing with respect to time 10 years from now?
(ii) At what percentage rate will the population be changing with respect to time t years from now?

Find the nth order differential coefficient of

$$y = \sin x \log(ax + b).$$

(7.75 + 6 + 5)

2. Convert the polar equation $r = 4\cos\theta + 6\sin\theta$ to rectangular coordinates. Show that it represents a circle. Find the centre and radius of that circle.

Identify and sketch the following conic by removing the xy-term

$$8x^2 - 12xy + 17y^2 = 20.$$

Find the equation of hyperbola with vertices $(0, \pm 3)$ and asymptotes $y = \pm x$.

(6 + 8.75 + 4)

3. Let R be the region bounded in the first quadrant by the curves $y = x^2$, the y-axis and the line $y = 1$. Determine the volume of the solid generated when R is revolved about the line $x = 2$ using cylindrical shell method and washer method.

Find the area of surface generated by the revolving the curves

(i) $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$ about x-axis,
(ii) $x = y^3$, $0 \le y \le 1$ about the y-axis.

(10.75 + 8)

4. Find the vector limit

$$\lim_{t\to 0+}\left[\left(1+\frac{1}{t}\right)^{t}\boldsymbol{i}-\left(\frac{\sin t}{t}\right)\boldsymbol{j}-\left(\frac{e^{-t}}{1-t}\right)\boldsymbol{k}\right].$$

A projectile is fired from ground level with muzzle speed 50 ft/s at an angle of elevation of $\alpha = 30^0$. What is the maximum height reached by the projectile? What is the time of flight and the range?

A particle moves along a path given in parametric form where $r(t) = 3 + 2\sin t$ and $\theta(t) = t^3$. Find the velocity and acceleration of the particle in terms of the unit polar vectors $\boldsymbol{u_r}$ and $\boldsymbol{u_\theta}$.

Find unit tangent $\boldsymbol{T}(t)$ and unit normal $\boldsymbol{N}(t)$ of the curve given by $r(t) = (t^2+1)\,\boldsymbol{i} + t\,\boldsymbol{j}$ at $t = 1$.

(3.75 + 5 + 5 + 5)

5. Let

$$L = \lim_{x\to \pi/2}(\sin x)^{\tan^2 x}$$

and

$$M = \lim_{x\to(\pi/2)-}(\tan x)^{\sin(2x)}.$$

Find the values of L and M and show that $eL^2 = M$.

Prove that $\cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right), x \geq 1$.

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$3x^2 + 4y^2 - 30x - 8y + 67 = 0.$$

(7.75 + 6 + 5)

6. If $y = \log(x+\sqrt{x^2+1})$, prove that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0.$$

The position of an object moving in space is given by

$$R(t) = (e^{-t}\cos t)\boldsymbol{i} + (e^{-t}\sin t)\boldsymbol{j} + e^{-t}\boldsymbol{k}.$$

Find the velocity, speed and acceleration of the object at arbitrary time t and at $t = 0$. Also, determine the curvature of the trajectory at arbitrary time t and at $t = 0$.

Prove that

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx.$$

Hence, evaluate $\int x^2 e^{3x} dx$.

(6 + 6 + 6.75)

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351102**
Name of Paper	: **BMATH102-Algebra**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find all the rational roots of the equation $224y^3 – 344y^2 + 22y – 15 = 0$ and also solve the equation $16y^4 - 96y^3 + 56y^2 + 264y – 135 = 0$ given that the roots form an arithmetical progression.
2. Draw a rough sketch of the region corresponding to the inequality $\frac{1}{\sqrt{2}} < |z - 1 - i| < \sqrt{2}$. Use De Movire's theorem to find the square root of $-3 + 4i$. Find the extended argument $Arg\, z$ of the complex number $z = (-\sqrt{3} - i)(1 + i)$.
3. Let $A = \{1,2,3,4,5,6,7\}$. Define a relation R_1 on the set A which is an equivalence relation. Define a relation R_2 on the set A which is not an equivalence relation. Let a be an integer, prove that there exists an integer k such that $a^2 = 5$k or $a^2 = 5k + 1$. Evaluate (5.6+8.11+19.23)(mod 9).
4. Show that the function $f: \left(\frac{2}{5}, \infty\right) \to \mathbb{R}$ and $g: \mathbb{R} \to \left(\frac{2}{5}, \infty\right)$ defined by $f(\text{x}) = log_5(5\text{x} - 2)$ and $g(x) = \frac{5^x+2}{5}$ are the inverse of each other. Prove that the interval (3, 7) and (1, ∞) have the same cardinality. Show that 314 and 159 are relatively prime integers.
5. Describe the solutions of the following system in parametric vector form. Give a geometrical description of the solution set.

$$4x_1 - 2x_2 + 6x_3 = 8$$
$$x_1 + x_2 - 3x_3 = -1$$
$$15x_1 - 3x_2 + 9x_3 = 21$$

Let T:$R^2 \to R^2$ be a linear transformation which first reflects points through the line $x_1 = x_2$ and then rotates points (about the origin) through π/4 radians. Find the standard matrix of T.

6. Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$. Find a basis for

(i) Row Space of A.

(ii) Null Space of A.

Also find Rank A and Nullity A.