[This question paper contains 4 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : **1381**

Unique Paper Code : 32351301

Name of the Paper : BMATH 305 – Theory of Real Functions

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any *two* parts from each question.
3. **All** questions are compulsory.

1. (a) Let $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A and $f: A \to \mathbb{R}$, then define limit of function f at c.

    Use $\varepsilon - \delta$ definition to show that $\lim_{x \to 1} \frac{x}{x+1} = \frac{1}{2}$.

    (6)

P.T.O.

(b) Let $f: A \to \mathbb{R}$, $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Then show that $\lim_{x \to c} f(x) = L$ if and only if for every sequence $\langle x_n \rangle$ in A that converges to c such that $x_n \neq c$, $\forall n \in \mathbb{R}$, the sequence $\langle f(x_n) \rangle$ converges to L. (6)

(c) Show that $\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$ does not exist in $\mathbb{R}$ but $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0$. (6)

2. (a) Let $A \subseteq \mathbb{R}$, $f: A \to \mathbb{R}$, $g: A \to \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Show that if f is bounded on a neighborhood of c and $\lim_{x \to c} g(x) = 0$, then $\lim_{x \to c} (fg)(x) = 0$. (6)

(b) Let $f(x) = e^{1/x}$ for $x \neq 0$, then find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0^+} f(x)$. (6)

(c) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 2x & \text{if } x \text{ is rational} \\ x + 3 & \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous. (6)

3. (a) Let $A \subseteq \mathbb{R}$ and let f and g be real valued functions on A. Show that if f and g are continuous on A then their product f g is continuous on A. Also, give examples of two functions f and g such that both are discontinuous at a point $c \in A$ but their product is continuous at c. (7½)

(b) State and prove Boundedness Theorem for continuous functions on a closed and bounded interval. (7½)

(c) State Maximum-Minimum Theorem. Let I = [a,b] and $f : I \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for each x in I. Prove that there exists a number $\alpha > 0$ such that $f(x) \geq \alpha$ for all x in I. (7½)

4. (a) Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ such that $f(x) \geq 0$ for all $x \in A$. Show that if f is continuous at $c \in A$, then $\sqrt{f}$ is continuous at c. (6)

(b) Show that every uniformly continuous function on $A \subseteq R$ is continuous on A. Is the converse true? Justify your answer. (6)

(c) Show that the function $f(x)=\frac{1}{x^2}$, $x \neq 0$ is uniformly continuous on $[a, \infty)$, for $a > 0$ but not uniformly continuous on $(0, \infty)$. (6)

5. (a) Let $I \subseteq \mathbb{R}$ be an interval, let $c \in I$, and let $f : I \to \mathbb{R}$ and $g : I \to \mathbb{R}$ be functions that are differentiable at c. Prove that if $g(c) \neq 0$, the function f/g is differentiable at n, and

$$\left(\frac{f}{g}\right)'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{(g(c))^2}.$$ (6)

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x| + |x + 1|$, $x \in \mathbb{R}$. Is f differentiable everywhere in $\mathbb{R}$? Find the derivative of f at the points where it is differentiable. (6)

(c) State Mean Value Theorem. If $f: [a, b] \to \mathbb{R}$ satisfies the conditions of Mean Value Theorem and $f'(x) = 0$ for all $x \in (a,b)$. Then prove that f is constant on [a, b]. (6)

6. (a) Let I be an open interval and let $f : I \to \mathbb{R}$ have a second derivative on I. Then show that f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. (6)

(b) Find the points of relative extrema of the functions $f(x) = |x^2 - 1|$, for $-4 \leq x \leq 4$. (6)

(c) Use Taylor's Theorem with n = 2 to approximate $\sqrt[3]{1+x}$, , $x > -1$. (6)

[This question paper contains 8 printed pages.]

(10)

Your Roll No...............

Sr. No. of Question Paper : 1409 **C**

Unique Paper Code : 32351302

Name of the Paper : BMATH306 – Group Theory-I

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question from **Q2** to Q6.

4. In the question paper, given notations have their usual meaning unless until stated otherwise.

P.T.O.

1. Give short answers to the following questions. Attempt any **six**.

(i) What is the total no of rotations and total no of reflections in the dihedral group D_3? Describe them (rotations and reflections) in pictures or words. What can you say about the total no of rotations and total no of reflections in the dihedral group D_n?

(ii) Give one non- trivial, proper subgroup of GL(2, R). Is GL(2, R) a group under addition of matrices? Answer in few lines.

(iii) Let G be a group with the property that for any a, b, c in G,

ab = ca implies b = c. Prove that G is Abelian.

(iv) Give an example of a cyclic group of order 5. Show that a group of order 5 is cyclic.

(v) Prove that a cyclic group is Abelian. Is the converse true?

(vi) Find all subgroups of Z_{15}.

(vii) Prove that 1 and -1 are the only two generators of (Z,+). Give short answer in few lines.

(viii) "Z_n. $n \in N$, is always cyclic whereas $U(n)$, $n \in N$; $n \geq 2$ may or may not be cyclic". Prove or disprove the statement in a few lines. $(6\times2=12)$

2. (a) Let $G = \{a + b\sqrt{2} \mid a$ and b are rational nos not both zero$\}$

Prove that G is a group under ordinary multiplication. Is it Abelian or Non-Abelian? Justify your answer.

(b) Prove that a group of composite order has a non-trivial, proper subgroup.

(c) Prove that order of a cyclic group is equal to the order of its generator.

(2×6.5=13)

3. (a) Prove that every permutation of a finite set can be written as a cycle or as a product of distinct cycles.

(6)

(b) (i) In S_4, write a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4.

(ii) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 8 & 6 & 7 & 5 & 1 & 3 \end{bmatrix}$ and

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 5 & 1 & 8 & 3 & 2 & 6 \end{bmatrix}$$

Write α, β and $\alpha\beta$ as product of 2-cycles. (3+3=6)

(c) (i) Let $|a| = 24$. How many left cosets of $H = \langle a^4 \rangle$ in $G = \langle a \rangle$ are there? Write each of them.

(ii) State Fermat's Little theorem. Also compute 5^{25} mod 7 and 11^{17} mod 7. (3+3=6)

4. (a) (i) Let H and K be two subgroups of a finite group. Prove that

$HK \leq G$ if G is Abelian.

(ii) Give an example of a group G and its two subgroups H and K ($H \neq K$) such that HK is not a subgroup of G. (3+3.5=6.5)

(b) (i) Let G be a group and let Z (G) be the centre of G. If G/Z (G) is cyclic, prove that G is Abelian.

(ii) Let $|G| = pq$, p and q are primes. Prove th
$|Z(G)| = 1$ or pq. (4+2.5=6.

(c) (i) Prove that a subgroup of index 2 i
normal.

(ii) Let $G = U(32)$, $H = U_8(32)$. Write all th
elements of the factor group G/H. Also fin
order of 3H in G/H. (3+3.5=6.5

5. (a) Show that the mapping from $\mathbb{R}$ under addition to 6.

$GL(2, \mathbb{R})$ that takes x to $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is a

group homomorphism. Also, find the kernel of the homomorphism.

(b) Let ϕ be a homomorphism from a group G to a group $\bar{G}$. Show that if $\bar{K}$ is a subgroup of $\bar{G}$, then $\phi^{-1}(\bar{K}) = [k \in G: \phi(k) \in \bar{K}\}$ is a subgroup of G.

(c) If H and K are two normal subgroups of a group G such that $H \subseteq X$, then prove that

$$G/K \approx \frac{G/H}{K/H}. \qquad (2\times6=12)$$

6. (a) Show that the mapping ϕ from $\mathbb{C}^*$ to $\mathbb{C}^*$ given by $\phi(z) = z^4$ is a homomorphism. Also find the set of all the elements that are mapped to 2.

(b) Prove that every group is isomorphic to a group of permutations.

(c) Let G be the group of non-zero complex numbers under multiplication and N be the set of complex numbers of absolute value 1.

P.T.O.

Show that G/N is isomorphic to the group of all the positive real numbers under multiplication.

(2×6.5=13)

[This question paper contains 4 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 1427

Unique Paper Code : 32351303

Name of the Paper : BMATH 307 – Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory
3. Attempt any **Five** questions from each section. **All** questions carry equal marks

SECTION I

1. Let $f(x,y) = \dfrac{xy\left(x^2 - y^2\right)x}{x^2 + y^2}$ if $(x, y) \neq (0,0)$

$= 0$ otherwise

Show that $f(0, y) = -y$ and $f(x, 0) = x$ for all x and y.

2. Use incremental approximation to estimate the function $f(x, y) = \sin(xy)$ at the point

$$\left(\sqrt{\frac{\pi}{2}}+.01, \sqrt{\frac{\pi}{2}}-.01\right)$$

3. If $z = xy + f(x^2 + y^2)$, show that $y\, \partial z/\partial x - x\, \partial z/\partial y = y^2 - x^2$.

4. Assume that maximum directional derivative of f at $P_0(1,2)$ is equal to 50 and is attained in the direction towards $Q(3, -4)$. Find ∇f at $P_0(1, 2)$.

5. Find the absolute extrema of $f(x, y) = 2x^2 - y^2$ on the disk $x^2 + y^2 \leq 1$.

6. Use Lagrange multiplier to find the distance from (0, 0, 0) to plane $Ax + By + Cz = D$ where at least one of A, B, C is nonzero.

SECTION II

1. Compute the integral $\int_0^1 \int_x^{2x} e^{y-x} dy\, dx$ with the order of integration reversed.

2. Use Polar double integral to show that a sphere of radius α has volume $\frac{4}{3}\pi a^3$.

3. Compute the area of region D bounded above by line $y = x$, and below by circle $x^2 + y^2 - 2y = 0$.

4. Find the volume of the solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

5. Evaluate $\iiint_D \frac{dx\,dy\,dz}{\sqrt{x^2 + y^2 + z^2}}$, where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

6. Use a suitable change of variables to find the area of region R bounded by the hyperbolas $xy=1$ and $xy=4$ and the lines $y=x$ and $y=4x$.

SECTION III

1. Find the mass of a wire in the shape of curve C: $x = 3 \sin t$, $y = 3 \cos t$, $z = 2t$ for $0 \leq t \leq \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = x$.

2. Find the work done by force

$$\vec{F}(x, y, z) = (y^2 - z^2)\hat{i} + (2yz)\hat{j} - (x^2)\hat{k}$$

on an object moving along the curve C given by $x(t) = t$, $y(t) = t^2$, $z(t) = t^3$, $0 \leq t \leq 1$.

3. Use Green's theorem to find the work done by the force field

$$\vec{F}(x,y) = (3y-4x)\hat{i} + (4x-y)\hat{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

4. Use Stoke's theorem to evaluate the surface integral

$$\iint_S \left(\text{curl } \vec{F}.N\right) dS$$

where $F = x\,i + y^2\,j + z\,e^{xy}\,k$ and S is that part of surface $z = 1 - x^2 - 2y^2$ with $z \geq 0$.

5. Use divergence theorem to evaluate the integral

$\iint_S \vec{F}.N\,dS$ where $\vec{F}(x,y,z) = (\cos yz)\hat{i} + e^{xz}\hat{j} + 3z^2\hat{k}$,

where S is hemisphere surface $z = \sqrt{4 - x^2 - y^2}$ together with the disk $x^2 + y^2 \leq 4$, in x-yplane.

6. Evaluate the line integral $\int_C \vec{F}.d\vec{R}$

Where $\vec{F}(x, y) = [(2x - x^2y)\,e^{-xy} + \tan^{-1} y]\,\hat{i} + \left[\frac{x}{y^2+1} - x^3 e^{-xy}\right]\hat{j}$ and C is the ellipse $9x^2 + 4y^2 = 36$.

(9)

[This question paper contains 8 printed pages.]

Your Roll No..............

C

Sr. No. of Question Paper : 1013

Unique Paper Code : 32351501

Name of the Paper : BMATH511 – Metric Spaces

Name of the Course : **B.Sc. (Hons.) Mathematics CBCS (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Let (X, d) be a metric space. Define the mapping $d^*: X \times X \to \mathbb{R}$ by

$$d^*(x,y) = \frac{d(x,y)}{1+d(x,y)};\ \forall x, y \in X.$$

P.T.O.

Show that (X, d^*) is a metric space and $d^*(x, y) < 1$, for every $x, y \in X$. (6)

(b) Let $\langle x_n \rangle_{n \geq 1}$ be a sequence of real numbers defined by $x_1 = a$, $x_2 = b$ and $x_{n+2} = \frac{1}{2}(x_{n+1} + x_n)$ for $n = 1, 2, \cdots$. Prove that $\langle x_n \rangle_{n \geq 1}$ is a Cauchy sequence in $\mathbb{R}$ with usual metric. (6)

(c) Define a complete metric space. Is the metric space $(\mathbb{Z}, d)$ of integers, with usual metric d, a complete metric space? Justify. (6)

2. (a) (i) Let (X, d) be a metric space. Show that for every pair of distinct points x and y of X, there exist disjoint open sets U and V such that $x \in U$, $y \in V$. (2)

(ii) Give an example of the following :

(a) A set in a metric space which is neither a closed ball nor an open set. (1)

(b) A metric space in which the interior of the intersection of an arbitrary family of the subsets may not be equal to the intersection of the interiors of the members of the family. (2)

(c) A metric space in which every singleton is an open set. (1)

(b) Let (X, d) be a metric space. Let A be a subset of X. Define closure of A and show that it is the smallest closed superset of A. (6)

(c) Let (X, d) be a complete metric space. Let $\langle F_n \rangle$ be a nested sequence of non-empty closed subsets of X such that $d(F_n) \rightarrow 0$. Show that $\bigcap_{n=1}^{\infty} F_n$ is a singleton. Does it hold if (X, d) is incomplete? Justify. (6)

3. (a) Let (X, d_X) and (Y, d_Y) be metric spaces and $f: X \rightarrow Y$ be a function. Prove that f is continuous on X if and only if $f(\bar{A}) \subseteq \overline{f(A)}$ for all subsets A of X. (6)

(b) Let A and B be non-empty disjoint closed subsets of a metric space (X, d). Show that there is a continuous real valued function f on X such that $f(x) = 0, \forall x \in A$, $f(x) = 1, \forall x \in B$ and $0 \leq f(x) \leq 1$, $\forall x \in X$. Further show that there exist disjoint open subsets G, H of X such that $A \subseteq G$ and $B \subseteq H$. (6)

(c) Define a dense subset of a metric space (X, d). Let $A \subseteq X$. Show that A is dense in X if and only if A^C has empty interior. Give an example of a metric space that has only one dense subset.

(6)

4. (a) Show that the metrics d_1, d_2 and d_∞ defined on $\mathbb{R}^n$ by

$$d_1(x,y) := \sum_{i=1}^{n} |x_i - y_i|,$$
$$d_2(x,y) = \left(\sum_{i=1}^{n}(x_i - y_i)^2\right)^{1/2} \text{ and}$$
$$d_\infty(x,y) = \max\{\,|x_i - y_i| : 1 \le i \le n\}$$

are equivalent where $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$. (6.5)

(b) Show that the function $f: \mathbb{R} \to (-1, 1)$ defined by

$f(x) = \dfrac{x}{1+|x|}$ is a homeomorphism but not an isometry. (6.5)

P.T.O.

(c) (i) Let (X, d) be a complete metric space. Let $T: X \to X$ be a mapping such that $d(Tx,Ty) < d(x, y)$, $\forall x, y \in X$. Does T always have a fixed point? Justify. (4)

(ii) Let X be any non-empty set and $T: X \to X$ be a mapping such that T^n (where n is a natural number, $n > 1$) has a unique fixed point $x_0 \in X$. Show that x_0 is also a unique fixed point of T. (2.5)

5. (a) Let $(\mathbb{R}, d)$ be the space of real numbers with usual metric. Prove that a connected subset of E must be an interval. Give an example of two connected subsets of E, such that their union is disconnected. (4+2.5)

(b) Let (X, d) be a metric space such that every two points of X are contained in some connected subset of X. Show that (X, d) is connected.

(6.5)

(c) Let (X, d) be a metric space. Then prove that (X, d) is disconnected if and only if there exists a continuous mapping of (X, d) onto the discrete two element space (X_0, d_0). (6.5)

6. (a) Prove that homeomorphism preserves compactness. Hence or otherwise show that

$$S(0,1) = \{z \in \mathbb{C} : |z| < 1\} \text{ and}$$

$$S[0,1] = \{z \in \mathbb{C} : |z| < 1\}$$

are not homeomorphic. (4+2.5)

(b) Let (X, d) be a metric space and $A \subseteq X$ such that every sequence in A has a subsequence converging in A. Show that for any $B \subseteq X$, there is a point $p \in A$ such that $d(p,B) = d(A,B)$. If B be a closed subset of X such that $A \cap B = \phi$, show that $d(A,B) > 0$. (4.5+2)

(c) Let f be a continuous real-valued function on a compact metric space (X, d_X), then show that f is bounded and attains its bounds. Does the result hold when X is not compact? Justify.

(4+2.5)

[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1049

Unique Paper Code : 32351502

Name of the Paper : BMATH512: Group Theory-II

Name of the Course : B.Sc. (H) Mathematics

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Question No. 1 has been divided into 10 parts and each part is of 1.5 marks.
4. Each question from Q. Nos. 2 to 6 has 3 parts and each part is of 6 marks. Attempt any two parts from each Question.

1. State true (T) or false (F). Justify your answer in brief.

   (i) A group of order p^2, p is a prime is always isomorphic to Z_{p^2}.

(ii) A group of order 15 is always cyclic.

(iii) A group of order 14 is simple.

(iv) The smallest positive integer n such that there are two non-isomorphic groups of order n is 6.

(v) Every inner automorphism induced by an element 'a' of group G is an automorphism of G.

(vi) A abelian group of order 12 must have an element of order either 2 or 3.

(vii) U(105) is isomorphic to external direct product of U(21) and U(5).

(viii) Center of a group G is always a subgroup of normalizer of A in G, where A is any subset of G.

(ix) $Aut(Z_{10})$ is a cyclic group of automorphisms of G.

(x) The largest possible order for an element of $Z_{20} \oplus Z_{30}$ is 60.

2. (a) Define inner automorphism induced by an element 'a' of group G and find the group of all inner automorphisms of D_4.

(b) Defile the characteristic and the commutator subgroup of a group. Prove that the centre of a group is characteristic subgroup of the group.

(c) Let G′ be the subgroup of commutators of a group G. Prove that G/G′ is abelian. Also, prove that if G/N is abelian, then $N \geq G'$.

3. (a) Determine the number of cyclic subgroups of order 15 in $Z_{90} \oplus Z_{36}$.

(b) Define the internal direct product of the subgroups H and K of a group G. Prove that every group of order p^2, where p is a prime, is isomorphic to Zp^2 or $Zp \oplus Zp$ (external direct product of Zp with itself).

(c) Consider the group G = {1, 9, 16, 22, 29, 53, 74, 79, 81} under multiplication modulo 91. Determine the isomorphism class of G.

4. (a) Show that the additive group Z acts on itself by z.a = z+a for all $z, a \in Z$.

(b) Show that an action is faithful if and only if its kernel is the identity subgroup.

(c) Let G be a group. Let H be a subgroup of G. Let G act by left multiplication on the set A of all left cosets of H in G. Let π_H be the permutation representation of G associated with this action. Prove tha,t

(i) G acts transitively on A

(ii) The stabilizer of the point $1H \in A$ is the subgroup H.

(iii) Ker $\pi_H = \cap_{x \in G} xHx^{-1}$

5. (a) Let G be a permutation group on a set A (G is subgroup of S_A), let $\sigma \in G$ and let $a \in A$. Prove that $\sigma G_a \sigma^{-1} = G_{\sigma(a)}$, here G_x denotes stabilizer of x. Deduce that if G acts transitively on A then $\cap_{x \in G} \sigma G_a \sigma^{-1} = 1$.

(b) Show that every group of order 56 has a proper nontrivial normal subgroup.

(c) State Index theorem and prove that a group of order 80 is not simple.

6. (a) State the Class Equation for a finite group G. and use it to prove that p-groups have non trivial centers.

(b) Prove that group of order 255 is always cyclic.

(c) Show that the alternating group As does not contain a subgroup of order 30, 20, or 15.

[This question paper contains 8 printed pages.]

Your Roll No....................

Sr. No. of Question Paper : 1132 **C**

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All **six** questions are compulsory.

3. Attempt any two parts from each question.

4. Use of non-programmable scientific calculator is allowed.

P.T.O.

1. (a) Define fixed point of a function and construct an algorithm to implement the fixed point iteration scheme to find a fixed point of a function. Find the fixed point of $f(x) = 2x(1 - x)$. (6)

(b) Perform four iterations of Newton's Raphson method to find the positive square root of 18. Take initial approximation $x_0=4$. (6)

(c) Find the root of the equation $x^3 - 2x - 6 = 0$ in the interval (2, 3) by the method of false position. Perform three iterations. (6)

2. (a) Define the order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Newton's iterative formula. (6.5)

(b) Find a root of the equation $x^3 - 4x - 8 = 0$ in the interval (2, 3) using the Bisection method till fourth iteration. (6.5)

(c) Perform three iterations of secant method to determine the location of the approximate root of the equation $x^3 + x^2 - 3x - 3 = 0$ on the interval (1, 2). Given the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. (6.5)

3. (a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that LU = PA

$$\text{where } A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{pmatrix}$$ (6.5)

(b) Set up the SOR method with w=0.7 to solve the system of equations:

$$3x_1 - x_2 + x_3 = 4$$

$$2x_1 - 6x_2 + 3x_3 = -13$$

P.T.O.

$$-9x_1 + 7x_2 - 20x_3 = 7$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Jacobi iteration scheme to solve the system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$

$$x_1 + 10x_2 - 5x_3 = -23$$

$$3x_1 - 2x_2 + 10x_3 = 38$$

Take the initial approximation as $X^{(0)} = (1, 1, 0)$ and do three iterations. (6.5)

4. (a) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data:

x	1	2	4	8
f(x)	3	7	21	73

(6)

(b) Calculate the Newton second order divided difference $\frac{1}{x^2}$ of based on the points x_0, x_1, x_2.

(6)

(c) Obtain the Lagrange form of the interpolating polynomial for the following data:

x	1	2	5
$f(x)$	-11	-23	1

(6)

5. (a) Find the highest degree of the polynomial for which the second order backward difference approximation for the first derivative

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}$$

provides the exact value of the derivative irrespective of h. (6)

(b) Derive second-order forward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)}{2h}.$$

(6)

(c) Approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ using the second order central difference formula taking $h = ½, ¼$ and $1/8$ and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the Simpson's rule, approximate the value of the integral $\int_2^5 \ln x \, dx$.. Verify that the theoretical error bound holds. (6.5)

(b) Apply Euler's method to approximate the solution of initial value problem $\frac{dx}{dt} = \frac{e^t}{x}, 0 \le t \le 2,$, $x(0) = 1$ and $N = 4$.

Given that the exact solution is $x(t) = \sqrt{2e^t - 1}$, , compute the absolute error at each step. (6.5)

(c) Apply the optimal RK2 method to approximate the solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $1 \le t \le 2$, $x(1) = 1$ taking the step size as $h = 0.5$. (6.5)

(10)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1133

Unique Paper Code : 32357502

Name of the Paper : DSE-1 Mathematical Modelling and Graph Theory

Name of the Course : **B.Sc. (H) Mathematics – CBCS (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) (i) Determine whether $x = 0$ is an ordinary point, a regular singular point or an irregular singular point of the differential equation

$$xy'' + x^3y' + (e^x - 1)y = 0. \qquad (6)$$

(ii) Find the Laplace transform of the function $f(t) = 1 + \cosh 5t$.

(iii) Find the inverse Laplace transform of the function $F(s) = \frac{1}{s+5}$.

(b) Use Laplace transforms to solve the initial value problem :
$x'' - x' - 2x = 0;\ x(0) = 0,\ x'(0) = 2$ (6)

(c) Find two linearly independent Frobenius series solutions of

$$2xy'' - y' - y - 0$$ (6)

(d) Find general solutions in powers of x of the differential equation. State the recurrence relation and the guaranteed radius of convergence.

$$5y'' - 2xy' + 10y = 0$$ (6)

2. (a) Using Monte Carlo simulation write an algorithm to compute volume of the surface $x^2 + y^2 + z^2 \le 1$ that lies in the first octant $x > 0,\ y > 0,\ z > 0$. (6)

(b) Use Simplex method to solve the given linear programming problem (6)

Maximize : $3x_1 + x_2$
subject to $2x_1 + x_2 \le 6$
$x_1 + 3x_2 \le 9$
$x_1, x_2 \ge 0.$

(c) Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unicading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	10	20	45	50	75
Unload time	70	35	40	80	90

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue and total time in which docking facilities are idle. (6)

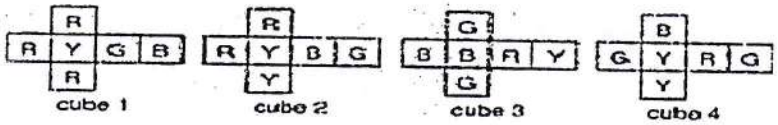
(d) Use Linear Congruence method to generate 15 random real numbers with multiplier 2, increment 5, modulus 13 and seed 1. Is there cycling? If yes, then give the period of cycling. (6)

3. (a) (i) Determine the number of edges of C_{16}, Q_3 and $K_{9,10}$. (3)

(ii) State and prove Handshaking Lemma. (3)

(b) Prove that a bipartite graph with odd number of vertices is not Hamiltonian. (6)

(c) Determine whether the given four cubes having four colours, can be stacked in a manner so that each side of the stack formed will have all the four colours exactly once. (6)



(d) By finding an Eulerian trail in K_5, arrange a set of fifteen dominoes [0 – 0 to 4 – 4] in a ring. (6)

4. (a) Use the factorization :

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s^3}{s^4 + 4a^4}\right\} = \cosh at \cos at$$ (7)

(b) Fit the model to the data using Chebyshev's criterion to minimize the largest deviation, given the model $y = cx$ and data set below :

y	1	2	3
x	2	5	8

(7)

(c) Solve the initial value problem using the Laplace transform

$x'' + 4x' + 13x = te^{-t}$; $x(0) = 0$, $x'(0) = 0$ (7)

(d) Name the five Platonic graphs. What is the degree of each vertex in each of these five graphs? Draw any two platonic graphs. (7)

[This question paper contains 12 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1134 C

Unique Paper Code : 32357503

Name of the Paper : DSE-1, C++ Programming for Mathematics

Name of the Course : **B.Sc. (Hons.) Mathematics, Part III (CBCS (LOCF))**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. This question paper has **six** questions in all.

3. Attempt any **two** parts from each question.

4. **All** questions are compulsory.

1. (a) (i) Write a C++ program to find the epsilon values of integer and real data types. (2)

(ii) Write down the C++ commands for finding the maximum and minimum values of four data types. (2)

(iii) Describe the difference between the increment and decrement operator. (2)

(b) (i) Write down the values of y, z and b, c in the following program

```
#include <iostream>
using namespace std;
int main() {
int x,y,z,a,b,c;
x=15;
y = + + x * 3.3-15 % 3;
z = z *z;
cout << y << endl << z << endl;
a=30;
b=a +2.2-Z*x % 3+ y--;
c = c/b;
cout << c << endl << b << endl;
return 0:
}
```

(2)

(ii) What are the header files for finding maximum and minimum size of integer and real data types? (1)

(iii) Write a program to input two complex numbers and find the sum, multiplication and division of two complex numbers using 'complex' header file. (3)

(c) If p_n is the probability that two integers chosen independently and uniformly from {1,2,...,n} are relatively prime. Write a program to calculate p_{15} by creating a header file gcd.h, which calculates the gcd of two numbers. (6)

2. (a) (i) Write the output of the following code in the matrix form. (4)

```
for (i = 0; i < 4; i++) {
for (j = 0; j < 4; j++)
{
if(i = = j)
A[i][j] = (i + j + 1)%2;
else
A[i][j] = (i*j+1)%2;
}
}
```

P.T.O.

(ii) Explain the difference between while and do while loop using an example in C++. (2)

(b) (i) Write a program which outputs the following pattern : (2)

1 2 3 4 5
1 2 3
1 2
1

(ii) Define a procedure which takes long type argument k to calculate and return the value of $\sum_{n=1}^{k}\frac{1}{n^3}$. Write a program which uses this procedure and displays the value of this expression for $k > 1$ entered by the user. (4)

(c) (i) Write the equivalent C++ expressions for the following :

(A) $y = 5(\cos x + \sin x)$; (B) $z = (x + y)^{z^2-5}$;

(C) $s = \frac{1}{200} gt^{30}$; (3)

(ii) Evaluate the following expression :

```
ini x = 1, y = 2;
    do {
    y++;
    ++x;

    cout<<x<<" "<< y << endl;

}   while ((x = = 3) || (y = = 3));
```

(3)

3. (a) Design the matrix class to represent an element of the group

$$M_2(Z_{11}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in Z_{11} \right\}$$

under component wise addition under modulo 11, having the following features :

(i) It should include four private data variables a, b, c, and d of double type. Why we are working on integers?

(ii) It should include the following two constructors :

(a) A default constructor to represent the zero matrix.

(b) A four argument constructor to represent the 2 by 2 real matrix.

(iii) It should include get methods to learn the values held by a, b, c, and d.

(iv) It should include an operator + method to add two objects of this class.

(v) It should include an operator << procedure for printing objects in the form $\begin{matrix} a & b \\ c & d \end{matrix}$.

Create any two objects A and B of this class to perform the addition A+B and display it to the screen. (6½)

(b) (i) What is the output of following program (3½)

```
#include <iostream>
    #include "Myprog.h"
    using namespace std;
    int main(){
        Myprog a;
        Myprog b(5);
```

```
cout<<a.get()<< " and "<<b.get();
return 0;}
```

Where associated header file Myprog.h is given by

```
#ifndef MYPROG_H
#define MYPROG_H
Class Myprog{
    Private: int x;
    Public: Myprog() {x=0;}
    Myprog(int y){x=y*y;}
    int get() const{retum x;}
    };
#endif
```

(ii) In the header file Myprog.h defined above, include an operator << procedure to print the object to the screen. (3)

(c) (i) Write a C++ program to generate pseudo random numbers in interval [0, 4) using LCG (linear congruential generator) defined as $x_{n+1} = (a\, x_n + b) \bmod c$, where a=17, b=3, c=64 and x_0=0. (4.5)

(ii) What does the rand() procedure return? Write the header file required for this. (2)

4. (a) Write a C++ procedure to find the Euler's totient function $\Phi(n)$. (6)

(b) (i) Write the syntax of the trigraph operator in C++. Explain it through an example. (2)

(ii) Explain function overloading with appropriate examples. (3)

(iii) Explain the inline procedures in C++. (1)

(c) (i) Write a C++ procedure to generate a random numbers in an arbitrary real interval [a, b]. (3)

(ii) Write a C++ program that prompts the user to input 15 positive integers and save them into an array. Perform the sorting on the array and print the sorted array to the screen. Program also display second largest element of the array. (3)

5. (a) Write a program to find the elements of U(10), where U(10) is the group under multiplication modulo 10 having elements less than and co-prime to 10, and store them in a Set A. The program then displays the element of U(10) and order of each element of U(10). (Order of an element x is the least positive integer n such that $x^n = 1$). (6½)

(b) What is the output of the following program.

```
#include <iostream>
#include <list>
using namespace std;
void print_list (list<long> & L1) {
list<long>::const_iterator Li;
for (Li = Ll.begin(); Li != Ll.end(); Li++) {
cout << *Li << " ";
}
cout << endl;
}
bool is_even(long n) {
return!(n%2 == 0);
}
```

```
int main() {
list<long> L;
L.insert(L.begin(),-5);
L.insert(L.end(),6);
L.insert (L.begin(),3);
L.push_front(2);
L.push_back(0);
L.push_front(-22);
list<long>:: iterator Li;
Li = L.begin();
Li++;
L.insert (Li,0);
print_list(L);
cout << L.front() << endl;
cont << L.hack() << endl;
cout << L.size() << endl;
L.sort();
print_list(L);
L.pop_front();
L.pop_back();
```

```
print_list(L);
L.remove_if(is_even);
print_list(L);
return 0;
}
```
(6½)

(c) (i) Write a program that reads numbers in a file till the end of the file, and displays the sum. Also, it should display a proper message if the file is empty. (4½)

(ii) Write the command to append data to an already exiting file on your computer, with required declaration. (2)

6. (a) (i) What is the difference between getline and get method in string class. (2)

(ii) What is the output of the following program code :

```
int main()
{
cout<<setw(10)<<left<<"Math"<<setw(7)<<showpos<<setfill('#')<<78<<endl;
cout<<setw(10)<<setfill('$')<<"Physics"<<setw(7)<<setfill(' @')<<50<<endl;
  return 0;
}
```
(2½)

(iii) In a class, what is a destructor and why we need it? (2)

(b) Write a procedure 'ReverseString' which takes a string as input and return reverse of it. Write main function which takes string from the user and display the reversed string using 'ReverseString' procedure and displays the length of the string. Also, it should compare the string entered by the user and reversed string. (6½)

(c) Write a program which takes two permutation maps f and g, using maps in C++, on the set {1,2,3,4.5,6} entered by the user. The program then finds the composition of these maps and display the output on the screen. (Composition of f and g is defined as: fog(x) = f(g(x))). Mention which composition you want to find out fog or gaf or both? (6½)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1231

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V (under CBCS (LOCF) Scheme)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the given **eight** questions are compulsory to attempt.
3. Do any **two** parts from each of the given **eight** questions.
4. Marks for each part are indicated on the right in brackets.

P.T.O.

SECTION I

1. (a) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as: For m, $n \in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \leq N_0$: $n = km$. Then show that $\leq$ is an order relation on N_0. (2½)

(b) If '1', '2', '3' denote chains of one, two, three elements respectively and $\overline{3}$ denotes anti chain of three elements, then draw the Hasse diagram for the dual of $L \oplus K$ when $L = \overline{3}$ and $K = 1 \oplus (2 \times 2)$. (2½)

(c) Define maximum and a maximal element of a partially ordered set P. Give an example each for both definitions. (2½)

2. (a) Let P and Q be finite ordered sets and let $\psi: P \rightarrow Q$ be a bijective map. Then show that the following are equivalent :

(i) $x < y$ in P iff $\psi(x) < \psi(y)$ in Q

(ii) $x \prec y$ in P iff $\psi(x) \prec \psi(y)$ in Q (3)

(b) Define upper bound and lower bound of a subset S of a partially ordered set P. Construct an example of a partially ordered set P and its subset S and give the set of all upper bounds and lower bounds of S. (3)

(c) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi: P \rightarrow Q$ and $\psi: Q \rightarrow P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \rightarrow S$ denotes the identity map on S given by: $id_S(x) = x$, $\forall x \in S$. (3)

SECTION II

3. (a) Let $D_{60} = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '$\leq$' is defined on D_{60} by $m \leq n$ if and only if m divides n then show that D_{60} does not form a lattice. Also Draw the diagram of D_{60} and find elements a, b, c, d $\in D_{60}$ such that $a \vee b$ and $c \wedge d$ do not exist in D_{60}. (5½)

(b) Define sublattice of a lattice. Prove that every chain of a lattice L is a lattice and also a sublattice of L. (5½)

(c) Define modular lattice. Prove that a homomorphic image of modular lattice is modular. (5½)

4. (a) Let L be a lattice. For any a, b, c $\in$ L, show that the following inequalities hold :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$.

(ii) $a \geq c \Rightarrow a \wedge (b \vee c) \geq (a \wedge b) \vee c$

(5)

(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. If we define

$$a \leq b : \Leftrightarrow a \vee b = b$$

then show that $(L, \leq)$ is a lattice ordered set. (5)

(c) Let L_1 and L_2 be distributive lattices. Prove that the product $L_1 \times L_2$ is a distributive lattice. (5)

SECTION III

5. (a) A voting-machine for three voters has YES-NO switches. Current is in the circuit precisely when YES has a majority. Draw the corresponding contact diagram and the switching/circuit diagram. (5½)

(b) Show that a Boolean Algebra is relatively complemented. (5½)

(c) Simplify the polynomial :

$$f = x'yz + x'yz' + x'y'z + xy'z' + xy'z$$

using Quine's McCluskey method. (5½)

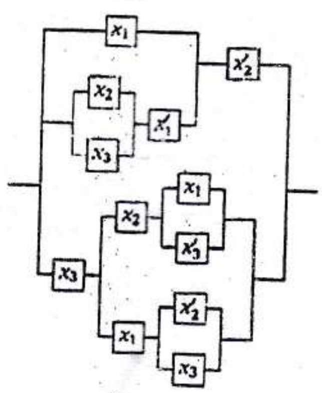
6. (a) Define a system of normal forms. Find conjunctive normal form for $p = y'z' + x'yz$. (5)

(b) Simplify the Boolean expression :

$$f = w'xy'z + w'xyz + w'xyz' + wxy'z + wxyz + wxyz' + wx'y'z + wx'yz$$

using Karnaugh Diagram. (5)

(c) Find the symbolic gate representation of the contact diagram : (5)



SECTION IV

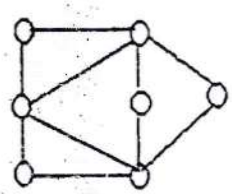
7. (a) (i) Show that the sum of the degrees of the vertices of a pseudograph is an even number equal to twice the number of edges.

(ii) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have? (5½)

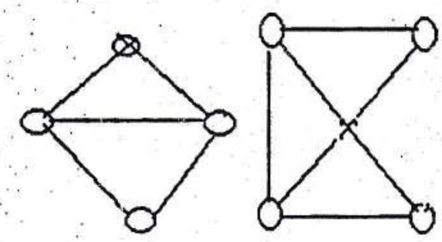
(b) (i) Define the degree sequence of a graph. Does there exist a graph with following degree sequence 6, 6, 5, 5, 4, 4, 4, 4, 3?

(ii) Show that the number of vertices of odd in a graph must be even. (5½)

(c) (i) What is a bipartite graph? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite.

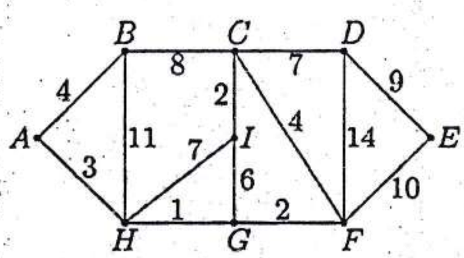


(ii) Define isomorphism of graphs. Also label the following graphs so as to show an isomorphism. (5½)



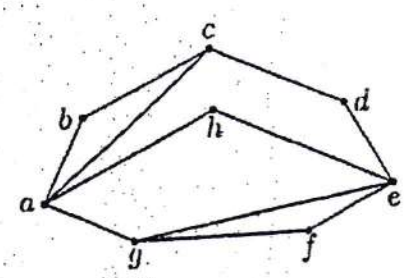
8. (a) Construct a Gray Code of length 3 using the concept of Hamiltonian Cycles. (5½)

(b) Apply Dijkstra's algorithm to find a shortest path from A to all other vertices in the weighted graph shown. (5½)



(c) (i) Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 6?

(ii) Define Eulerian circuit. Is the given graph Eulerian? Give reasons for your answer.



(5½)