[This question paper contains 5 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7462** **J**

Unique Paper Code : 32351102 - OC

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Algebra

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) Attempt any **two** parts from each questions.

(iii) **All** questions are compulsory.

1. (a) Find the polar representation for the complex number 6

$z = 1 - \text{Cos}a + i\,\text{Sin}a, \quad a \in [0, 2\pi)$

(b) Solve the equation $(2 - 3i)z^6 + 1 + 5i = 0$. 6

(c) Compute $z^n + \frac{1}{z^n}$, if $z + \frac{1}{z} = \sqrt{3}$. 6

P.T.O.

2. (a) Define ~ on $\mathbb{Z}$ by a~b if and only if 2a + 3b = 5n for some integer n. Prove that ~ defines an equivalence relation on $\mathbb{Z}$. 6

(b) Define $f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 3x^3 - x$.

(i) Is f one-to-one ?

(ii) Is f onto ?

Justify each answer. 6

(c) Show that the open intervals (0, 1) and (1, 2) have the same cardinality. 6

3. (a) Define relatively prime integers. Show that 17,369 and 5,472 are relatively prime. Hence, find integers x and y such that 17369x + 5472y = 1. 6

(b) (i) Show that $3^6 \equiv 1 \pmod 7$ and hence evaluate $3^{60} \pmod 7$.

(ii) Find all integers x(mod12) that satisfy $9x \equiv 3 \pmod{12}$. 6

(c) Use the Principle of Mathematical Induction to prove $2^{2n} - 1$ is divisible by 3, $\forall n \geq 1$. 6

4. (a) Write the solution set of the given system of equations in parametric vector form. 6.5

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_2 - 6x_3 = -3$$

(b) Let $A = \begin{pmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{pmatrix}$. Show that the equation Ax = b may not be consistent for every $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Also describe the set of all vectors b for which Ax = b is consistent. 6.5

(c) Determine h and k such that the solution set of the given system 6.5

$$x_1 + 3x_2 = k$$

$$4x_1 + h x_2 = 8$$

(i) is empty.

(ii) contains a unique solution.

(iii) contains infinitely many solutions.

5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is $B_2S_3 + H_2O \to H_3BO_3 + H_2S$.

Balance the chemical equation using the vector equation approach. 6.5

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8597**

Unique Paper Code : **32351101** **J**

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

Attempt any *four* questions from Section I.

1. State Leibnitz's theorem for finding *n*th derivative of product of two functions. If $y = a\cos(\ln x) + b\sin(\ln x)$, prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

P.T.O.

2. Evaluate the following limit :

$$\lim_{x \to 0^+} x^{\sin x}.$$

3. Find the intervals of increase and decrease of the following function, discuss its concavity and then sketch its graph $y = (x+1)^2(x-5)$.

4. Sketch the graph of the polar curve $r = 3\cos 2\theta$.

5. A manufacturer estimates that when 'x' units of a particular commodity are produced each month, the total cost (in dollars) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that corresponds to the maximum profit.

Section II

Attempt any *four* questions from Section II.

6. Find a reduction formula for $\int \operatorname{cosec}^n x\, dx$, $n \geq 2$ is an integer. Evaluate $\int \operatorname{cosec}^4 x\, dx$.

7. Find the volume of the solid generated when the region bounded by $y = \sqrt{25 - x^2}$, $y = 3$, is revolved about the x-axis.

8. The base of a certain solid is enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross-section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.

9. Find the arc length of the parametric curve :

$$x = (1 + t)^2,\ y = (1 + t)^3,\ 0 \leq t \leq 1.$$

10. Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, about the x-axis.

Section III

Attempt any *three* questions from Section III.

11. Find the equation of the parabola whose focus is $(-1, 4)$ and directrix is $x = 5$.

12. Find the equation of the hyperbola whose foci are $(1, 8)$ and $(1, -12)$ and vertices are 4 units apart.

13. Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$

14. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

Section IV

Attempt any *four* questions from Section IV.

15. Evaluate :

$$\lim_{t \to 0^+} [\frac{\sin 3t}{\sin 2t}\hat{i} + \frac{\log(\sin t)}{\log(\tan t)}\hat{j} + (t \log t)\hat{k}].$$

16. The acceleration of a moving particle is $\vec{A}(t) = 24t^2\hat{i} + 4\hat{j}$. Find the particle's position as a function of t if $\vec{R}(0) = \hat{i} + 2\hat{j}$ and $\vec{v}(0) = 0$.

17. If a shot putter throws a shot from a height of 5 ft with an angle of 46° and initial speed of 25 ft/sec, what is the horizontal distance of the throw ?

18. Find $\vec{T}(t)$, $\vec{N}(t)$ and $\vec{B}(t)$ for $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$ at $t = \frac{\pi}{4}$.

19. Show that the curvature of the polar curve $r = e^{\alpha\theta}$ is inversely proportional to r.

[This question paper contains 4 printed pages]

Your Roll No.	: ....
Sl. No. of Q. Paper	: **7461** **J**
Unique Paper Code	: 32351101 - OC
Name of the Course	: **B.Sc.(Hons.) Mathematics**
Name of the Paper	: Calculus
Semester	: I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** the sections are compulsory.

(c) All questions carry equal marks.

(d) Use of non-programmable scientific calculator is allowed.

Section-I

Note : Attempt any **four** questions from this **Section**.

1. If $y = \sin^{-1} x$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n = 0$.

2. Sketch the graph of $f(x) = \frac{1}{x+1} + \frac{1}{x-1}$ by finding intervals of increase and decrease, critical points, points of relative maxima and minima, concavity of the graph and inflection points.

P.T.O.

3. Evaluate analytically following problem :

$\lim_{x\to\infty}\left(x\sin^{-1}\left(\frac{1}{x}\right)\right)^x$

4. Suppose a manufacturer estimates that, when the market price of a certain product is p, the number of units sold will be $= -6\ln\left(\frac{p}{40}\right)$. It is also estimated that the cost of producing these *x* units will be $C(x) = 4xe^{-\frac{x}{6}} + 30$.

(a) Find the average cost, the marginal cost, and the marginal revenue for this production process.

(b) What level of production *x* corresponds to maximum profit ?

5. Sketch the graph of the curve in polar coordinates $r = 4 - 4\cos\theta$.

Section - II

Note : Attempt any **four** questions from this **Section**.

6. Find the reduction formula for $\int x^n e^x dx$ and hence evaluate $= \int_0^1 xe^{-\sqrt{x}}dx$.

7. Find the volume of the solid generated when the region enclosed by the curves $y = \sqrt{25 - x^2}$ and y = 3, is revolved about x-axis.

8. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curve $y = \frac{1}{x^3}, x = 1, x = 2, y = 0$ is revolved about the line x = –1.

9. Find the exact arc length of the curve $y = \frac{x^6 + 8}{16x^2}$ from x = 2 to x = 3.

10. Find the area of the surface generated by revolving the curve $x = \sqrt{9 - y^2},\ -2 \le y \le 2$. about y-axis.

Section- III

Note : Attempt any **three** questions from this **Section**.

11. State the reflection properties of the conic sections : parabolas, ellipses and hyperbolas with diagram.

12. Find an equation for the parabola that has its vertex at (1,2) and its focus at (4,2).

13. Describe the graph of the equation $9x^2 + 4y^2 + 18x - 24y + 9 = 0$ with rough sketch label the foci, vertices and the ends of minor axis.

14. Trace the conic $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$ by rotating the coordinate axes to remove the xy term.

Section - IV

Note : Attempt any **four** questions from this **Section**.

15. Find the position vector and velocity vector if acceleration vector with initial conditions are given as $A(t)=(\cos t)\hat{i}-(t\sin t)\hat{k}; R(0)=\hat{i}-2\hat{j}+\hat{k}$; $V(0)=2\hat{i}+\hat{k}$.

16. A boy standing at the edge of a cliff throws a ball upward at a 30° angle with an intial speed of 64 ft/s. Suppose that when the ball leaves the boy's hand, it is 48 ft above the ground at the base of the cliff.

(a) What are the time of flight of the ball and its range ?

(b) What are the velocity of the ball and its speed at impact ?

(c) What is the highest point reached by the ball during its flight ?

17. Find the tangential and normal components of acceleration of an object that moves with position vector $R(t)=(\sin t)\hat{i}+(\cos t)\hat{j}+(\sin t)\hat{k}$.

18. An object moves along the curve in the plane described in polar form $r = 3+2\sin t$; $\theta = t^3$.

Find its velocity and acceleration in terms of unit polars U_r and U_θ.

19. Find the curvature and radius of curvature at the stated point for a curve

$x = e^t \cos t, \quad y = e^t \sin t, \quad z = et \quad t = 0$

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7464** **J**

Unique Paper Code : 32351302

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Group Theory - I

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) All questions carry equal marks.

1. (a) Let $G = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in R, a \neq 0 \right\}$. Show that G is a group under matrix multiplication.

(b) Let G be a group and H be a subset of G. Prove that H is a subgroup of G if $a, b \in H \Rightarrow ab^{-1} \in H$. Hence prove that H = {$A \in G$: det A is a power of 3} is a subgroup of GL (2, R).

P.T.O.

(c) (i) Suppose G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have ?

(ii) If $|a| = n$ and k divides n, prove that $|a^{n/k}|=k$.

6 × 2 = 12

2. (a) Let G = <a> be a cyclic group of order n. Prove that G = $<a^k>$ if and only if gcd (k, n) =1. List all the generators of Z_{20}.

(b) (i) If a cyclic group has an element of infinite order, how many elements of finite order does it have.

(ii) List all the elements of order 6 and 8 in Z_{30}.

(c) Suppose that a and b are group elements that commute and have orders m and n. If $<a> \cap <b> = \{e\}$, Prove that the group contains an element whose order is the least common multiple of m and n. Show that this need not be true if a and b do not commute.

6.5 × 2 = 13

3. (a) Let G be a group. Is $H = \{x^2 : x \in G\}$ a subgroup of G ? Justify.

(b) Prove that any two left cosets of a subgroup H in a group G are either equal or disjoint.

(c) Show that (Q ,+) has no proper subgroup of finite index. 6 × 2 = 12

4. (a) Prove that every subgroup of index 2 is normal. Show that A_5 is normal subgroup of S_5.

(b) Let G be a group and H be a normal subgroup of G. Prove that the set of all left cosets of H in G forms a group under the operation aH.bH = abH where a, b ∈ G.

(c) If H is a normal subgroup of G with |H| =2, prove that $H \subseteq Z(G)$. Hence or otherwise show that A_5 cannot have a normal subgroup of order 2. 6.5 × 2 = 13

5. (a) Let C be the set of complex numbers and

$$M = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in R \right\}.$$

Prove that C and M are isomorphic under addition and $C^* = C \setminus \{0\}$ and $M^* = M \setminus \{0\}$ are isomorphic under multiplication.

(b) Prove that a finite cyclic group of order n is isomorphic to the group $Z_n = \{0, 1, 2, \dots, n-1\}$ under addition modulo *n*.

(c) (i) Suppose that φ is an isomorphism from a group G onto a group G*. Prove that G is cyclic if and only if G* is cyclic.

(ii) Show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition. $6 \times 2 = 12$

6. (a) Let φ be a group homomorphism from a group G to a group G^* then prove that :

(i) $|\varphi(x)|$ divides $|x|$, for all x in G.

(ii) φ is one-one if and only if $|\varphi(x)| = |x|$, for all x in G.

(b) State and prove the Third Isomorphism Theorem.

(c) (i) Let G be a group. Prove that the mapping $\phi(g) = g^{-1}$, for all $g \in G$, is an isomorphism on G if and only if G is Abelian.

(ii) Determine all homomorphisms from Z_n to itself. $6.5 \times 2 = 13$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7279 **J**

Unique Paper Code : 32353301

Name of the Paper : Latex and HTML

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : III

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.

1. Fill in the blanks **(Any 4)** : (4×½=2)

   (i) To create a hyperlink in HTML .... element is used.

   (ii) LaTeX is a .... language.

   (iii) The command .... draws a circle with center (2,2) and radius 1.

(iv) Boldface text on a webpage is obtained with the .... element.

(v) The command to produce name of institute in a beamer presentation is ....

2. Answer any **eight** parts from the following : (8×2=16)

(i) Describe three different ways in LaTeX to write in math mode.

(ii) What is wrong with the following input:

$$\$theta = pi\$, \text{then } \$\sin theta = 0\$.$$

(iii) What is the output of the following command :

\[\left(\frac{a+b}{x+y}\right)^{1/3}\]

(iv) Make the following equation in LaTex:

$$R_\theta = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

(v) Give any two attributes of the img tag in HTML.

(vi) Typeset a code in LaTeX for the following :

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

(vii) Give the output of the command

\psarc(1,1){3}{0}{50}

(viii) Write a LaTeX code to produce $p^q + q^p + z^z$ as the output.

(ix) Write the output of the following HTML code :

<h3> Ordered list with Arabic numerals </h3>

<o1 type = "1">

 Analysis

 Algebra

</o1>

(x) Write the postfix notation in standard form: x sin 1 add 2 exp 1 x sub div.

3. Answer any **five** parts from the following : (5×4=20)

(i) Write a code in LaTeX for typesetting the following expression:

$$A_n = \begin{bmatrix} n & n^2 & n^3 \\ 3 & 9 & 27 \\ 4 & 16 & 64 \\ 11 & 121 & 1331 \end{bmatrix}$$

(ii) Find the errors in the following LaTeX source, write a corrected version and write its output :

```
\documentclass{article}

\usepackage{amsmath}

\title{My Document}

\author{ABC}

\date{today}

\maketitle

\begin{document}

\[ lim_{n \rightarrow infinity} \frac{sin 2 x}{x} \]

\end{document}
```

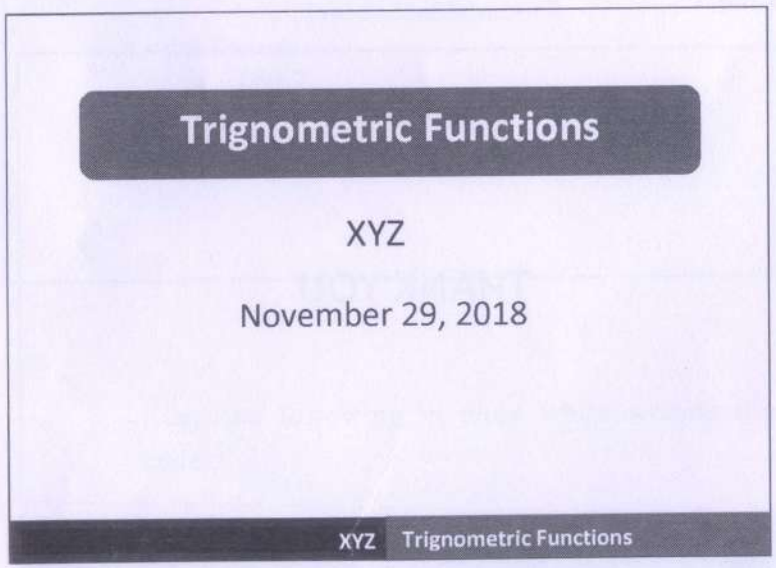
(iii) Write the code in LaTeX to plot the functions $y=\sqrt{x}$ and $y=x^2$ on the same coordinate system, for $0 \le x \le 1$. Show the sine function as a solid curve and the cosine function as a dotted curve.

(iv) Write a code in LaTeX for typesetting the following expression :

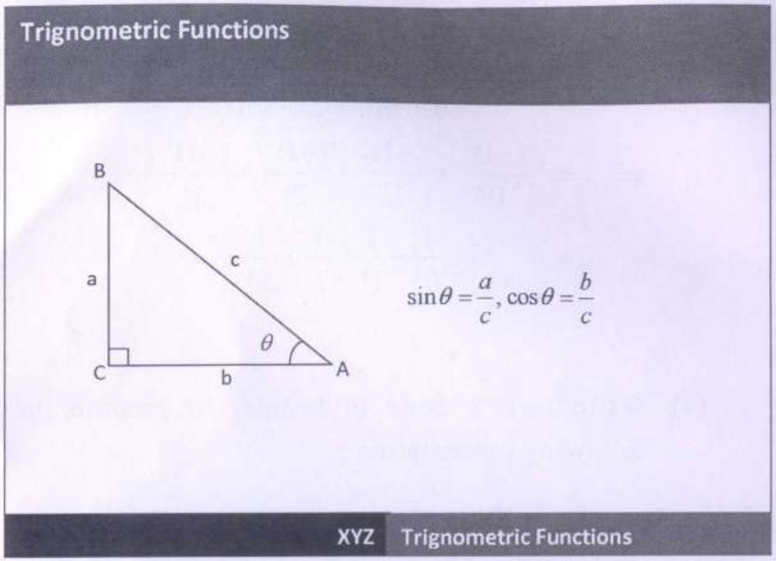
$$\begin{aligned} e^x &= \frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \\ e^{-1} &= \frac{(-1)^0}{0!}+\frac{(-1)^1}{1!}+\frac{(-1)^2}{2!}+\frac{(-1)^3}{3!}+\cdots \\ &= \frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots \end{aligned}$$

(v) Write LaTeX code in beamer to prepare the following presentation :

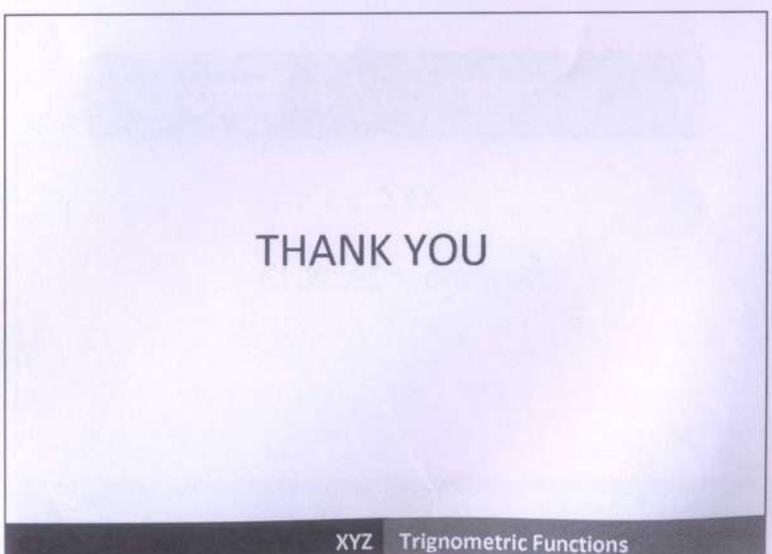
Slide 1:



Slide 2:



Slide 3:



(vi) Write an HTML code to generate the following web page:

University of Delhi

Department of Mathematics

The list of options for DSE papers offered in B.Sc.(H)-Mathematics:

1. Vth Semester
   a. DSE-1
      i. Numerical Methods
      ii. Mathematical Modelling and Graph Theory
   b. DSE-2
      i. Mathematical Finance
      ii. Discrete Mathematics
2. VIth Semester
   a. DSE-3
      i. Probability Theory & Statistics
      ii. Mechanics

Keep the following in mind while writing the code :

(i) Font face of the text should be Arial.

(ii) Text color of the main heading should be purple.

[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7465 J**

Unique Paper Code : 32351303

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Multivariate Calculus

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** Sections are compulsory.

(iii) Attempt any **five** questions from each **Section**.

(iv) All questions carry equal marks.

P.T.O.

Section - II

7. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

8. Sketch the region of integration and then compute the integral $\int_0^1 \int_x^{2x} e^{y-x}\, dy\, dx$ in 2 ways :

(a) with the given order of integration

(b) with the order of integration reversed

9. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$ by converting to polar coordinates.

10. Find the volume of the tetrahedron bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0$, $y = 0$ and $z = 0$.

11. Compute $\iiint_D \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$ where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

12. Use the change of variables to compute $\iint_D \frac{(x-y)^4}{(x+y)^4}\, dydx$, where D is the triangular region bounded by the line $x + y = 1$ and the coordinate axes.

Section - III

13. Find the work done by the force field $\vec{F} = \frac{x}{\sqrt{x^2+y^2}}\,\bar{i} - \frac{y}{\sqrt{x^2+y^2}}\,\bar{j}$ when an object moves from (a,0) to (0,a) on the path $x^2 + y^2 = a^2$.

14. Verify that the following line integral is independent of the path $\oint (3x^2 + 2x + y^2)\, dx + (2xy + y^3)\, dy$ where C is any path from (0,0) to (0,1).

15. Use Green's theorem to evaluate $\oint_C \left(x \sin x dx - \exp\left(y^2\right) dy\right)$ where C is the closed curve joining the points (1,-1) (2,5) and (-1,-1) in counterclockwise direction.

16. State Stoke's theorem and use it to evaluate $\iint_S \text{curl}\vec{F}.dS$ where $\vec{F} = xz\vec{i} + yz\vec{j} + xy\vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

17. Use the divergence theorem to evaluate the surface integral $\iint_S \vec{F}.\vec{N}dS$, where $\vec{F} = \left(x^2 + y^2 - z^2\right)\vec{i} + yx^2\vec{j} + 3z\vec{k}$; S is the surface comprised of the five faces of the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, missing $z = 0$.

18. Evaluate $\iint_S 2x dS$ where S is the portion of the plane $x + y + z = 1$ with $x \geq 0, y \geq 0, z \geq 0$.

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[This question paper contains 4 printed pages]

Your Roll No.	: ....
Sl. No. of Q. Paper	: **7463 J**
Unique Paper Code	: 32351301
Name of the Course	: **B.Sc.(Hons.) Mathematics**
Name of the Paper	: Theory of Real Functions
Semester	: III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **three** parts from each question.

(c) **All** questions carry equal marks.

1. (a) Find the following limit and establish it by using $\in-\delta$ definition of limit :

$$\lim_{x \to -1} \frac{x+5}{2x+3}$$

(b) State and prove the sequential criterion for limits of a real valued function.

P.T.O.

(c) Determine whether the following limit exists in R :

$$\lim_{x\to 0} \operatorname{sgn}\left(\sin 1/x^2\right)$$

(d) Show that :

$$\lim_{x\to 0^-} e^{\frac{1}{x}} = 0$$

and establish that

$$\lim_{x\to 0^+} e^{\frac{1}{x}}$$

does not exist in R .

2. (a) Let $c \in R$ and f be defined on (c, ∞) and $f(x) > 0$ for all $x \in (c, \infty)$. Show that

$$\lim_{x\to\infty} f(x) = \infty$$

if and only if

$$\lim_{x\to\infty} \frac{1}{f(x)} = 0$$

(b) Evaluate the following limit by using the appropriate definition :

$$\lim_{x\to 1^+} \frac{x}{x-1}$$

(c) Determine the points of continuity of the function $f(x) = x[x]$ where [.] denotes the greatest integer function.

(d) A function $f: R \to R$ is such that $f(x+y) = f(x) + f(y)$ for all x,y in R. Prove that if f is continuous at some point x_0, then it is continuous at every point of R.

3. (a) Let $A \subseteq R$ and $f: A \to R$ and let $f(x) \geq 0$, for all $x \in A$. Let $\sqrt{f}$ be defined as $\sqrt{f}(x) = \sqrt{f(x)}$ for $x \in A$. Show that if f is continuous at a point $c \in A$, then $\sqrt{f}$ is continuous at c.

(b) Suppose that $f: R \to R$ is continuous on R and that $f(r) = 0$ for every rational number r. Prove that $f(x) = 0$ for all $x \in R$.

(c) Let f be a continuous and real valued function defined on a closed and bounded interval [a, b]. Prove that f is bounded. Give an example to show that the condition of boundedness of the interval cannot be dropped.

(d) State the intermediate value theorem. Show that $x_2^k = 1$ for some $x \in]0, 1[$.

4. (a) Show that the function $f(x) = x^2$ is uniformly continuous on $[-2, 2]$, but it is not uniformly continuous on *R*.

(b) Prove that if f and g are uniformly continuous on $A \subseteq R$ and if they both are bounded on A, then their product fg is uniformly continuous on A.

(c) Show that the function

$f(x) = |x + 1| + |x - 1|$

is not differentiable at -1 and 1.

(d) Prove that if $f : R \to R$ is an even function and has a derivative at every point, then the derivative f' is an odd function.

5. (a) State Darboux theorem. Let I be an interval and $f : I \to R$ be differentiable on I. Show that if the derivative f' is never zero on *I*, then either $f'(x) > 0$ for all $x \in I$ or $f'(x) < 0$ for all $x \in I$.

(b) Find the Taylor's series for cos x and indicate why it converges to cos x for all $x \in R$.

(c) Prove that $e^x \geq 1 + x$ for all $x \in R$, with equality occurring if and only if $x = 0.5$.

(d) Is $f(x) = |x|$, $x \in R$, a convex function ? Is every convex function differentiable ? Justify your answer.

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8086**

Unique Paper Code : **32357506** **J**

Name of the Paper : **Cryptography and Network Security**

Name of the Course : **B.Sc. (Hons) Mathematics : DSE-1**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *five* parts from question No. **1**, each part carries **3** marks.

Attempt any *two* parts from questions **2** to **6**, each part carries **6** marks.

1. (*a*) Use the Rail fence cipher of depth 3 to encrypt "there could be better questions." Which attack is this cipher vulnerable to ?

(*b*) Explain the term diffusion in the context of a block cipher. How does DES achieve diffusion ?

(*c*) What is the difference between a stream cipher and a block cipher ?

P.T.O.

(*d*) Describe a trap-door-one-way function.

(*e*) Define Euler totient function ϕ. Compute ϕ (105).

(*f*) Write the order in which Compression, Encryption and Digital Signatures are applied in PGP, while achieving both Authentication and Confidentiality, clearly state the reason behind this order.

(*g*) Describe the terms Non-Repudiation and Integrity in context of Cryptography. Mention the cryptographic primitives used to achieve these.

2. (*a*) Decrypt the following message encrypted using playfair cipher with the key "HER MAJESTY'S SHIP".

LVHZ CRJE RQQO ZRTY ERGM JRRM XOJR RANF RMOW ODNM AHYN WDER NMFM.

(*b*) What does it mean to say that the one time pad is unbreakable ? If the one time pad is unconditionally secure, why is it not widely used ?

(*c*) Describe the key expansion algorithm of DES with the help of a diagram.

3. (*a*) For any positive integers a and n, show that $b \equiv c$ $mod(n)$ implies $ab \equiv ac$ $mod(n)$. Show that converse is not true in general. In which case converse is also true ?

(*b*) Determine the GCD of $x^4 + 2x^3 + 5x^2 + 5x + 4$ and $x^3 + 2x^2 + 3x + 6$ over GF(7).

(*c*) State Fermat's Theorem. Use Fermat's Theorem to reduce 8^{109} (mod 37).

4. (*a*) Describe the general structure of the encryption process in AES with the help of a diagram. Briefly comment on the various transformations performed in each round.

(*b*) Suppose that we have the following 128-bit AES key, given in haxadecimal notation :

287E151628AED2A6ABF7158809CF4F3C

(*i*) Express the initial round key (w_0 w_1 w_2 w_3) as a State matrix.

(*ii*) Given that RC[1] = 01, S(09) = 01, S(CF) = 8A, S(4F) = 84 and S(3C) = EB, where S denotes the S-box, calculate the first four bytes (w_4) for round one.

(c) Define the discrete logarithm of a number b for the base a (*mod p*). Prove that : $dlog_{a,p}\ (xy) = [dlog_{a,p}\ (x) + dlog_{a,p}\ (y)]\ (mod\ \phi(p))$.

5. (a) Perform encryption and decryption using the RSA algorithm for $p = 7$, $q = 13$, $e = 5$ and M = 8.

(b) The public parameters of Alice consists of an elliptic curve $y^2 = x^3 + x + 6$ over the field GF(11) and a point G = (2, 7) on this curve. Suppose Alice's private key is $a = 2$. Bob sends the ciphertext ((8,3), (5,9)) to Alice. Find the message sent by Bob to Alice.

(c) For the elliptic curve $y^2 = x^3 + x + 6$ over the field GF(11) :

(i) Calculate P + Q, where P = (5,2), Q = (8, 3).

(ii) Calculate 2P, where P = (5,2).

6. (a) What is the maximum input size and length of output of hash function SHA-512. State the value of padding field and length fields if the message length is 1920 bits. What is the size of word (register used) in SHA-512 ?

(b) Alice uses Flgamal Digital signature scheme to sign a document with the parameters : A cyclic group GF(19) with generator $a = 10$ and private key X = 16. He generated the random K = 5, gcd (K, 18) = 1 as part of signing process. If Alice signed the document with hash value $m = 14$, claculate the signature.

(c) Define Digital Signatures, its parameters, input/output, and general working of signing and verification algorithms. Define three types of attacks on a Digital signature.

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8085**

Unique Paper Code : **32357505** **J**

Name of the Paper : **Discrete Mathematics**

Name of the Course : **B.Sc. (Hons.) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *two* parts from each question.

SECTION I

1. (*a*) Define an order isomorphism for two ordered sets P and Q. Show that an order isomorphism is a bijection, but the converse is not true. 6

(*b*) Let P and Q be ordered sets. Prove that :

$(a_1, b_1) \prec (a_2, b_2)$ in $P \times Q$ iff ($a_1 = a_2$ and $b_1 \prec b_2$) or ($a_1 \prec a_2$ and $b_1 = b_2$). 6

(*c*) Let P, Q and R be ordered sets and let $\varphi : P \to Q$ and $\psi : Q \to R$ be order preserving maps. Then show that the composite map : $\psi o \varphi : P \to R$ given by :

$$(\psi o \varphi)(x) = \psi(\varphi(x)) \text{ for } x \in P,$$

is also an order preserving map. 6

2. (*a*) For a lattice L with operations $\vee$ and $\wedge$, show that $\forall\ a, b, c \in L$:

(*i*) $a \vee b = b \vee a$

(*ii*) $a \vee (a \wedge b) = a$

(*iii*) $(a \vee b) \vee c = a \vee (b \vee c)$. 6½

(*b*) Let L and K be lattices and $f : L \to K$ be a map. Then show that the following are equivalent :

(*i*) f is order preserving

(*ii*) $(\forall\ a, b, \in L)\ f(a \wedge b) \leq f(a) \wedge f(b)$. 6½

(*c*) (*i*) Let L be a lattice and let $a, b, c \in L$. Then show that :

$$b \leq a \leq (b \vee c) \text{ implies } a \vee c = b \vee c.$$

(*ii*) Is the set of real numbers with usual order, a lattice ? Is it complete ? Give reasons for your answer. 6½

SECTION II

3. (*a*) Prove that if L and K are modular lattices then their product is also a modular lattice. 6

(*b*) (*i*) Prove that every distributive lattice is modular lattice. Is the converse true ? Justify your answer. 3

(*ii*) Prove that in a distributive lattice complement of any element, if it exists, is unique. 3

(*c*) Find the disjunctive normal form of $f(x, y, z) = (x \vee y) \wedge (x \vee y') \wedge (x' \vee z)$. 6

4. (*a*) For all x, y in a Boolean Algebra, prove that :

(*i*) $(x \vee y)' = x' \wedge y'$

(*ii*) $x \leq y \Leftrightarrow x \wedge y' = 0 \Leftrightarrow x' \vee y = 1 \Leftrightarrow x \wedge y = x \Leftrightarrow x \vee y = y$. 6½

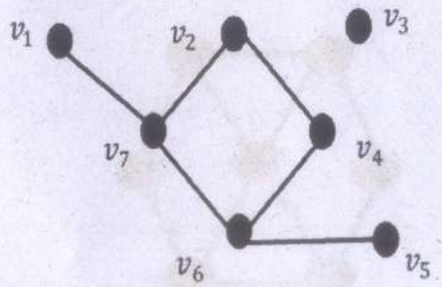
(*b*) Find all prime implicants of $xy'z + x'yz' + xyz' + xyz$ and form the corresponding Prime implicant table. 6½

(*c*) Give the symbolic representation of the following Boolean polynomial and then simplify : 6½

$$f = (x_1x_3'x_4') + (x_1x_2'x_4) + (x_1x_3x_4')$$

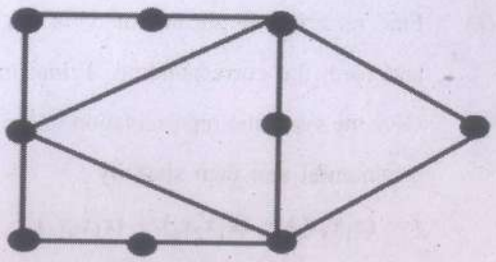
SECTION III

5. (*a*) (*i*) Find the degree sequence of the graph in the figure below. Verify that the sum of vertex degrees is an even number. Also, verify that the number of odd vertices is even :

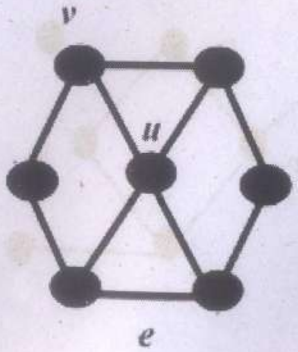


(*ii*) Find the number of edges in the complete graph $K_{3,6}$. 4+2

(*b*) (*i*) Determine if the graph in the figure below is Bipartite. If so, find the partition sets or justify why the graph is not bipartite.

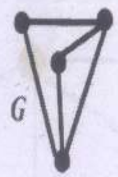


(*ii*) For the graph G in the figure below, draw the subgraphs G | {*e*} and G | {*v*}.



3+3

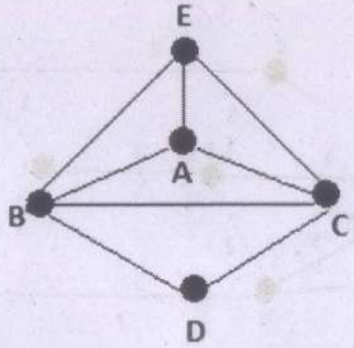
(*c*) (*i*) Label the graphs G and H suitably to show that they are isomorphic :



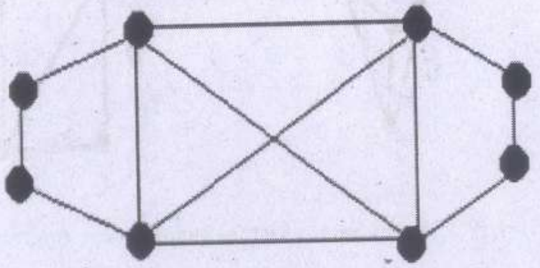
(*ii*) Does there exist a graph with degree sequence 4, 3, 2, 1. Justify your answer.

(*iii*) Draw the complete graphs K_4 and $K_{3,3}$. 2+2+2

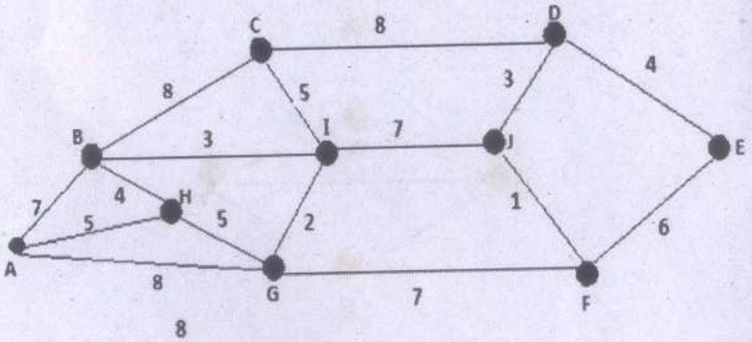
6. (*a*) (*i*) Define a Hamiltonian graph. Is the graph given below Hamiltonian. Explain.



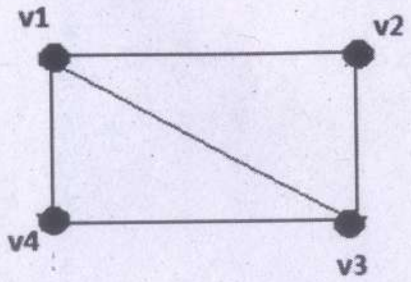
(*ii*) Describe an Eulerian circuit by numbering the edges of the graph. 3½,3



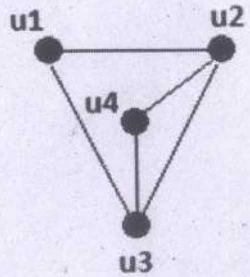
(*b*) Apply the Dijkstra's algorithm to find the length of the shortest path from A to E shown below : 6½



(*c*) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 as shown below. Find a permutation matrix P such that $A_2 = PA_1 P^T$: 6½



G_1



G_2

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7945**

Unique Paper Code : **32357505** **J**

Name of the Paper : **Discrete Mathematics**

Name of the Course : **B.Sc. (Hons.) Mathematics : DSE-1**

Semester : **V**

Duration : 3 **Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *two* parts from each question.

SECTION I

1\. (*a*) (*i*) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as :

For $m, n \in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \in N_0 : n = km$, then show that $\leq$ is an order relation on N_0.

(*ii*) Draw Hasse diagram for the subset $P = \{1, 2, 3, 12, 18, 0\}$ of $(N_0; \leq)$, where $\leq$ same as defined above. 3+3 (*b*) Show that two finite ordered sets P and Q are order isomorphic iff they can be drawn with identical diagrams. 6

(*c*) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi : P \to Q$ and $\psi : Q \to P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \to S$ denotes the identity map on S given by : $id_S(x) = x$, $\forall x \in S$. 6

2. (*a*) Let $(L, \wedge, \vee)$ be a non-empty set equipped with two binary operations $\wedge$ and $\vee$. Also L is such that the following laws, associative law, commutative law, idempotency law and absorption law and their duals hold. Then show that :

(*i*) $(a \vee b) = b$ iff $(a \wedge b) = a (\forall a, b \in L)$

(*ii*) Define a relation $\leq$ on L as $a \leq b$ if $(a \vee b) = b$. Then prove that $\leq$ is an order relation on L. 6.5

(*b*) Let L and K be lattices and $f : L \to K$ be a map. Then show that the following are equivalent :

(*i*) f is order preserving

(*ii*) $(\forall a, b, \in L)$ $f(a \vee b) \geq f(a) \vee f(b)$. 6.5

(*c*) Prove that in any lattice L, we have :

$((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y$ 6.5

$(\forall x, y, z \in L)$.

SECTION II

3. (*a*) Let L be a lattice. Prove that L is distributive if and only if for all elements a, b, c of L,

$(a \vee b = c \vee b$ and $a \wedge b = c \wedge b)$ implies $a = c$. 6

(*b*) Find the conjunctive normal form of $f = (x(y' + z)) + z'$ in three variables. Also find its disjunctive normal form. 6

(*c*) Prove that every Boolean algebra is sectionally complemented. 6

4. (*a*) Find the prime implicants of $xy + xy'z + x'y'z$ and form the corresponding prime implicant table. 6.5

(*b*) Simplify the following function using the Karnaugh diagram : 6.5

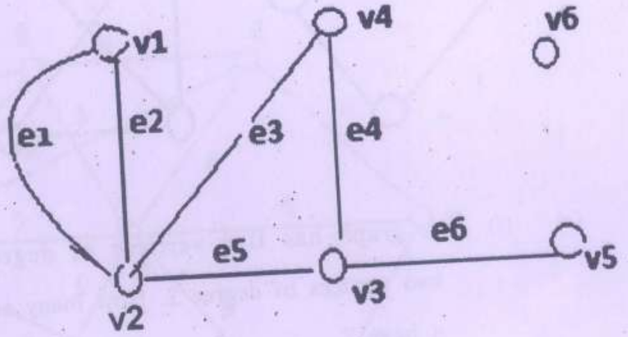
$$x_1x_2x'_3 + x'_1x_2x'_3 + (x_1 + x'_2x'_3)(x_1 + x_2 + x_3)' + x_3(x'_1 + x_2).$$

(*c*) A motor is supplied by three generators. The operation of each generator is monitored by a corresponding switching element which closes a circuit as soon as generator fails. In the electrical monitoring system, a warning lamp lights up if one or two generators fail. Determine a symbolic representation as a mathematical model of this problem. 6.5

SECTION III

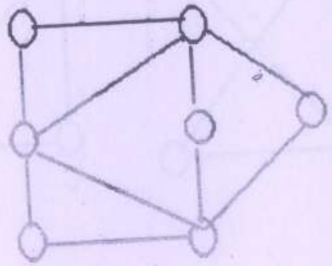
5. (*a*) (*i*) Prove that number of odd vertices in a pseudo graph is even.

(*ii*) Find the degree sequence for G; verify that the sum of the degrees of the vertices is an even number. Which vertices are even ? Which are odd ?

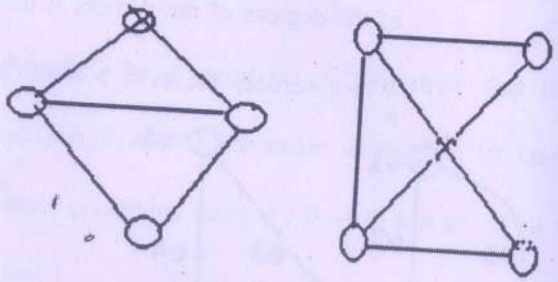


2+4

(*b*) (*i*) What is bipartite graph ? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite.



(ii) Define isomorphism of graph. Also label the graphs so as to show an isomorphism.

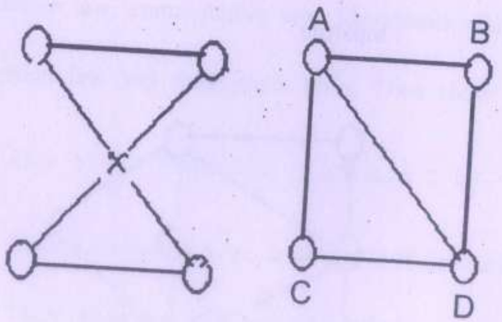


3+3

(c) (i) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have ?

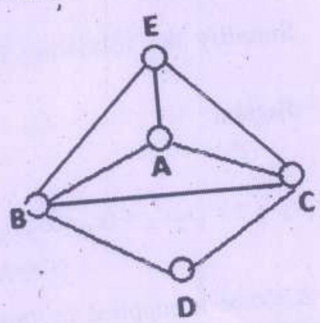
(ii) Why can there not exist a graph whose degree sequence is 5, 4, 4, 3, 2, 1.

(iii) Explain why the graphs are not isomorphic.



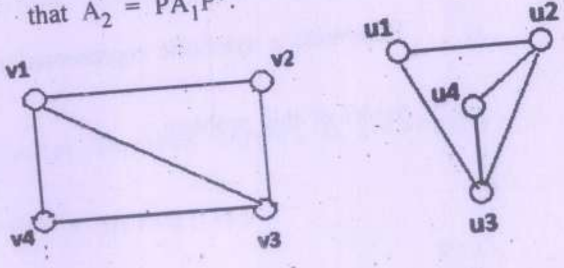
2+2+2

6. (a) (i) Define Hamiltonian graph. Is the graph given below Hamiltonian ? If no, explain. If yes, find a Hamiltonian cycle.



(ii) Answer the Konisberg bridge problem and explain. 6.5

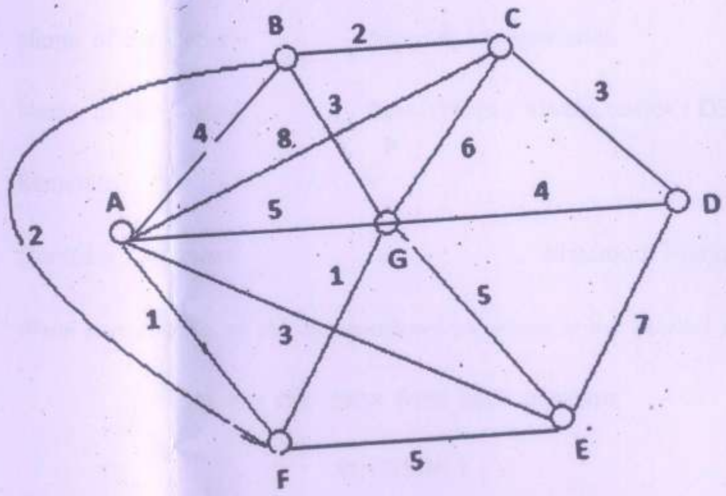
(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 as shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.



G_1 G_2

6.5

(c) Apply the improved version of Dijkstra's algorithm to find the length of a shortest path from A to D in the graph shown below. Write steps.



6.5

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8081**

Unique Paper Code : **32357501** **J**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Use of Non-Programmable Scientific Calculator is allowed.

1. (*a*) A real root of the equation $x^3 - 5x + 1 = 0$ lies in]0, 1[. Perform three iterations of Regula Falsi Method to obtain the root. 6

P.T.O.

(*b*) Perform three iteration of Bisection Method to obtain root of the equation $\cos(x) - xe^x$ in $]0, 1[$. 6

(*c*) Discuss the order of convergence of the Secant method and give the geometrical interpretation of the method. 6

2. (*a*) Verify $x = \sqrt{a}$ is a fixed point of the function $h(x) = \frac{1}{2}\left(x + \frac{a}{x^2}\right)$. Determine order of convergence of sequence $p_n = h(p_{n-1})$ towards $x = \sqrt{a}$. 6.5

(*b*) Use Secant method to find root of $3x + \sin(x) - e^x = 0$ in $]0, 1[$. Perform three iterations. 6.5

(*c*) Prove that Newton's Method is of order two using $x^3 + 2x^2 - 3x - 1 = 0$ and initial approximation $x_0 = 2$. 6.5

3. (*a*) Define a lower and an upper triangular matrix. Solve the system of equations :

$$-3x_1 + 2x_2 - x_3 = -12$$

$$6x_1 + 8x_2 + x_3 = 1$$

$$4x_1 + 2x_2 + 7x_3 = 1$$

by obtaining an LU decomposition of the coefficient matrix A of the above system. 6

(*b*) For Jacobi method, calculate T_{jac}, C_{jac} and spectral radius of the following matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$ 6

(*c*) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$4x_1 + 2x_2 - x_3 = 1$$

$$2x_1 + 4x_2 + x_3 = -1$$

$$-x_1 + x_2 + 4x_3 = 1$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. 6

4. (*a*) Construct the Lagrange form of the interpolating polynomial from the following data :

x	1	2	3
$f(x) = \ln x$	ln 1	ln 2	ln 3

6.5

(*b*) Prove that for $n + 1$ distinct nodal points $x_0, x_1, x_2, \ldots, x_n$, there exists a unique interpolating polynomial of at most degree n. 6.5

(*c*) Find the maximum value of the step size h that can be used in the tabulation of $f(x) = e^x$ on the interval [0, 1] so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}. 6.5

5. (*a*) Define the backward difference operator ∇ and the Newton divided difference. Prove that :

$$f[x_0, x_1, ------, x_n] = \frac{\nabla^n f_n}{n! h^n} \text{ where } h = x_{i+1} - x_i .$$ 6

(*b*) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial :

x	–7	–5	–4	–1
y	10	5	2	10

Find the approximation of y for $x = -3$. 6

(*c*) Use the formula

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

to approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 1$ taking $h = 1, 0.1, 0.001$. What is the order of approximation ? 6

6. (*a*) Approximate the value of $\int_0^1 e^{-x} dx$ using the Trapezoidal rule and verify that the theoretical error bound holds for the same. 6.5

(*b*) State Simpson's 1/3rd rule for the evaluation of $\int_a^b f(x) dx$ and prove that it has degree of precision 3. 6.5

(*c*) Use Euler's method to approximate the solution of the initial value problem.

$x' = (1 + x^2)/t$, $x(1) = 0$, $1 \le t \le 4$ taking 5 steps. 6.5